I

a) $f(x) = x^3 - x^2 + x - 2$ $m = 4$ $[0, 3]$

$f(0) = -2$ $f(3) = 27 - 9 + 3 - 2 = 19$

$-2 < 4 < 19$

$x^3 - x^2 + x - 2 \overset{\text{set}}{=} 4$

$x^3 - x^2 + x - 6 = 0$ E is integer in $[0, 3]$

$8 - 4 + 2 - 6 = 0$

$0 = 0$ $E = 2$

b) $f(x) = x^2 - 6x + 8$ $m = 0$ $[0, 3]$

$f(0) = 8$ $f(3) = 9 - 18 + 8 = -1$

$-1 < 0 < 8$

$x^2 - 6x + 8 \overset{\text{set}}{=} 0$

$(x - 4)(x - 2) = 0$

$x \in \{4, 2\}$. only $x = 2$ in interval

$E = 2$

c) $f(x) = x^2 + x - 1$ $m = 11$ $[0, 5]$

$f(0) = -1$ $f(5) = 25 + 5 - 1 = 29$

$-1 < 0 < 29$ $x^2 + x - 1 \overset{\text{set}}{=} 11$

$x^2 + x - 12 = 0$

$(x + 4)(x - 3) = 0$

$x \in \{-4, 3\}$.

$E = 3$ only one in interval.

d) $f(x) = \dfrac{x^2 + x}{x - 1}$ $m = 1$ $\left[\frac{5}{2}, 4\right]$

$f\left(\frac{5}{2}\right) = \dfrac{\frac{25}{4} + \frac{5}{2}}{\frac{5}{2} - 1}$ $f(4) = \frac{16 + 4}{3} = \frac{20}{3}$

$\frac{35}{4} \cdot \left(\frac{2}{3}\right)$ invalid

$\frac{35}{6}$ not between $\frac{35}{6}$ and $\frac{20}{3}$.

e) $f(x) = \dfrac{x^2 + 2x + 1}{x + 1}$ $m = -\frac{1}{2}$ $[-2, 2]$

$= \dfrac{(x+1)^2}{x+1}$ invalid.

$= x + 1$ f not continuous in $[-2, 2]$ 3 pts

f) $f(x) = \dfrac{x^2 - 1}{x^2 + 1}$ $m = 2$ $[0, 3]$

$f(0) = -1$ $f(3) = \frac{8}{10} = \frac{4}{5}$

N.T valid 2 not between -1 and $\frac{4}{5}$

g) (1) $m = 2$ $[-1, 8]$ Not valid

f not continuous on $[1, 8]$

(4) $m = 0$ $[1, 8]$

Not valid

0 not between 3 and 6

(2) $m = 2$ $[1, 8]$

$f(1) = 3$ $f(8) = 6$

invalid 2 not between 3 and 6.

(3) $m = 5$ $[1, 8]$

$f(1) = 3$ $f(8) = 6$

$E = 7$

II $f(x) = \begin{cases} x^2 + bx + c & x \le 1 \\ x + 1 & 1 < x < 3 \\ x^2 + bx + c & x \ge 3 \end{cases}$

$f(1) = 1 + b + c$

$\lim_{x \to 1^-} f(x) = 1 + b + c$

$\lim_{x \to 1^+} 2$

$\therefore 1 + b + c = 2$

$b + c = 1$

$f(3) = 9 + 3b + c$

$\lim_{x \to 3^-} 4$

$\lim_{x \to 3^+} 9 + 3b + c$

$\therefore 4 = 9 + 3b + c$

$3b + c = -5$

$-b - c = -1$

$2b = -6$

$b = -3$

$\therefore c = 4$

III

a) $f(x) = \frac{x+1}{x-1}$

$f(x+h) = \frac{x+h+1}{x+h-1}$

$f(x+h) - f(x) = \frac{x+h+1}{x+h-1} - \frac{x+1}{x-1}$

$\frac{x^2+hx+x-x-h-1}{x^2+hx-h-1} - \left[\frac{x^2+hx-x+x+h-1}{}\right]$

$x^2+hx-h-1 - x^2-hx+h-1$

$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$\lim_{h\to 0} \frac{1}{h}\left[\frac{-2h}{(x+h-1)(x-1)}\right]$

$\frac{-2}{(x-1)^2}$

b) $f(x) = \cos x$

$\lim_{h\to 0} \frac{\cos(x+h)-\cos x}{h}$

$\lim_{h\to 0} \frac{\cos x\cos h - \sin x \sin h - \cos x}{h} =$

$\lim_{h\to 0} -\cos x\left(\frac{1-\cos h}{h}\right) - \frac{\sin h}{h}\cdot \sin x =$

$-\cos x(0) - 1 \sin x =$

$f'(x) = -\sin x.$

IV a) $\lim_{x\to 0} \frac{\sin x}{x}$

$\lim_{x\to 0} \frac{\sin x}{x}\cdot\frac{1}{\cos x}$

$1 \cdot \frac{1}{1} = 1$

b) $\lim_{x\to -1} \frac{1-|x|}{1+x}$

$\lim_{x\to -1} \frac{1+x}{1+x}$

1

$|x| = -x$ since $x<0$

c) $\lim_{x\to 0} \frac{(1-\cos^2 x)^2 + \sin^2 3x}{x^2}$

$\lim_{x\to 0} \frac{\sin^4 x + \sin^2 3x}{x^2}$

$\lim_{x\to 0} \frac{\sin^2 x}{x^2}\cdot\sin^2 x + \frac{\sin^2 3x}{9x^2}\cdot 9$

$1\cdot 0 + 1\cdot 9$

9

d) $\lim_{x\to 0} \frac{\sin 2x}{\sin x}$

$\lim_{x\to 0} \frac{2\sin x\cos x}{\sin x}$

$\lim_{x\to 0} 2\cos x = 2.$

Extra

$\sqrt{5+\sqrt{x}} = \sqrt{x}-1$

$5+\sqrt{x} = x - 2\sqrt{x}+1$

$3\sqrt{x} = x-4$

$9x = x^2 - 8x + 16$

$x^2 - 17x + 16 = 0$

$(x-16)(x-1) = 0$

$x \in \{1, 16\}.$

must check for extraneous root.

$\sqrt{5+1} = 1-1$

$\sqrt{6} = 0$

Reject $x=1$

$\sqrt{5+4} = \sqrt{16}-1$

$\sqrt{9} = 4-1$

$3 = 3$

good root

$x = 16.$

2000

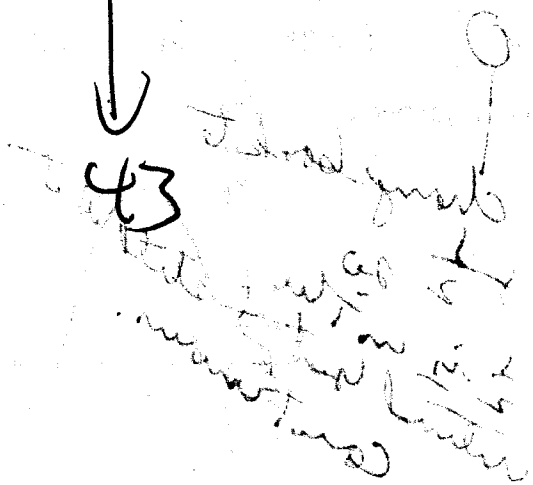
100	2
90	2
80	9
70	3
60	9
59	3

102

43

2960
27

76.3



Several were Not prepared for LUT.

I guess they will be now....

I guess may mid apt for make up ~~ex~~ fun. we'll see.

one change on this exam as noted on other side.

10/8/21

excellent
~~bfm~~.

100	0
90	4
80	5
70	3
60	2
59	2

94

50

$$\frac{1260}{16} = 78.75$$

change I instructions as indicated.

change IE from $m=2$ to $m=-\frac{1}{2}$

~~$f(-2)=-1$~~

$f(2)=\frac{1}{3}$ so the m will be between f(a) and f(b)

10/13/95

100	1
90	6
80	4
70	6
60	2
59	11

104

55

Put some of the IUT Problems on the multiple choice Sheets.

Next year won't this Problem because we have this test post out talking years don't get enough Practice on these.

1643

20

82.15

I guess the 104 on 6th 99 Practice on this Exam from a cache of old exams

A.P. Calculus
Copy original problem.
Convince *me* that **you** understand the concept!
No calculators, of course.

Name Calc 2nd Exam
Per _____ Date 2006

Exam #2 Quarter #1 Chapters 1 – 4

I Find the value of X in the 4 situations (of 10) where the Intermediate Value Theorem can be properly applied.
Note: ***X is an integer*** in each valid case. Explain your reasoning for the invalid cases.

(5 pts ea – total 50 pts)

A) $f(x)=x^3-x^2+x-2$; $m=4$ $[0,3]$

B) $f(x)=x^2-6x+8$; $m=0$ $[0,3]$

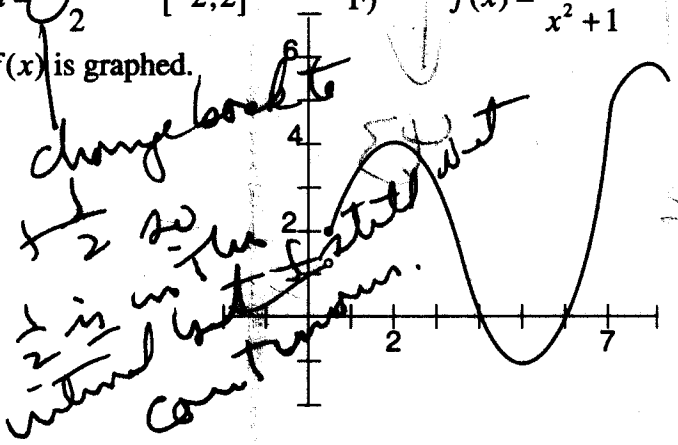
C) $f(x)=x^2+x-1$; $m=11$ $[0,5]$

D) $f(x)=\dfrac{x^2+x}{x-1}$; $m=1$ $\left[\dfrac{5}{2},4\right]$

E) $f(x)=\dfrac{x^2+2x+1}{x+1}$; $m=-\dfrac{1}{2}$ $[-2,2]$

F) $f(x)=\dfrac{x^2-1}{x^2+1}$; $m=2$ $[0,3]$

G) See figure to the right. $f(x)$ is graphed.



1) $m=2$ $[-1,8]$
2) $m=2$ $[1,8]$
3) $m=5$ $[1,8]$
4) $m=0$ $[1,8]$

II Determine the values of the letters b and c so the function is continuous, if possible. (10 pts)

$$F(x)=\begin{cases} x^2+bx+c & x\le 1 \\ x+1 & 1<x<3 \\ x^2+bx+c & x\ge 3 \end{cases}$$

III Using the definition of the derivative, find the derivative of $f(x)$: (10 pts ea)

A) $f(x)=\dfrac{x+1}{x-1}$ B) $f(x)=\cos x$

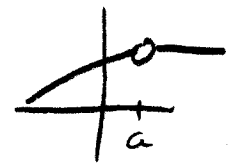
IV Determine the following limits, if possible. Briefly explain your reasoning. (5 pts ea)

A) $\lim\limits_{x\to 0}\dfrac{\tan x}{x}$ B) $\lim\limits_{x\to -1}\dfrac{1-|x|}{1+x}$ C) $\lim\limits_{x\to 0}\dfrac{\left(1-\cos^2 x\right)^2+\sin^2 3x}{x^2}$ D) $\lim\limits_{x\to 0}\dfrac{\sin 2x}{\sin x}$

Extra Credit 5 pts

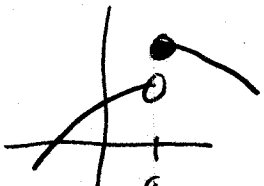
Solve $\forall\ x\varepsilon\Re$: $\sqrt{5+\sqrt{x}}=\sqrt{x}-1$

I



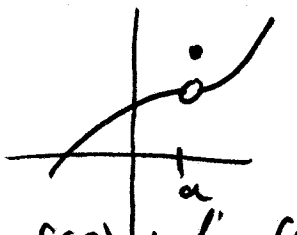
$f(a)$ DNE

b)



$\lim_{x\to a} f(x)$ DNE

c)



$f(a) \neq \lim_{x\to a} f(x)$

II

$$\frac{1+\sin x}{1-2\sin x} = \frac{3}{4}$$

$$4+4\sin x = 3-6\sin x$$

$$10\sin x = -1$$

$$\sin x = -\frac{1}{10}$$

$$\sin^2 x = \frac{1}{100} \quad \therefore \cos^2 x = \frac{99}{100}$$

$$\cos 2x =$$

$$\cos^2 x - \sin^2 x$$

$$\frac{99}{100} - \frac{1}{100}$$

$$\frac{98}{100}$$

$$\frac{49}{50}$$

III

$$f(x) = \begin{cases} x^2 & x \le 1 \\ 2x-1 & x > 1 \end{cases}$$

a) $f(1) = 1$ ① ← ③ ≡ ② ↓

$\lim_{x\to 1^-} x^2 = 1$

$\lim_{x\to 1^+} 2x-1 = 1$

] $\lim_{x\to 1} f(x) = 1$

b) $\lim_{x\to 1} \frac{f(x)-1}{x-1}$

$\lim_{x\to 1^-} \frac{x^2-1}{x-1}$ $\lim_{x\to 1^-} x+1 = 2$

$\lim_{x\to 1^+} \frac{2x-1-1}{x-1}$ $\lim_{x\to 1^+} \frac{2(x-1)}{x-1}$ 2

$\lim_{x\to 1} \frac{f(x)-1}{x-1} = 2$

$\therefore f'(1)$ exists and is equal to 2.

IV

a) $\lim_{x\to 2} \frac{\sqrt{x^2+5}-3}{x-2} \cdot \frac{\sqrt{\ }+3}{\sqrt{\ }+3}$

$\lim_{x\to 2} \frac{x^2+5-9}{(x-2)(\sqrt{x^2+5}+3)}$

$\lim_{x\to 2} \frac{x+2}{\sqrt{x^2+5}+3}$

$\frac{4}{3+3}$

$\frac{2}{3}$

b) $\lim_{h\to 0} \left(\sqrt{1+\frac{1}{x}} - \sqrt{\frac{1}{x}}\right)$

$\lim_{h\to 0} \frac{\sqrt{x+1}-1}{\sqrt{x}} \cdot \frac{\sqrt{\ }+1}{\sqrt{\ }+1}$

$\lim_{h\to 0} \frac{x+1-1}{\sqrt{x}(\sqrt{x+1}+1)}$

$\lim_{h\to 0} \frac{\sqrt{x}}{\sqrt{x+1}+1} = \frac{0}{2} = 0$

IV c $\lim_{x\to 1}\left[\frac{1}{1-x^3}-\frac{1}{3(1-x)}\right]$

$\lim_{x\to 1}\left[\frac{3-(1+x+x^2)}{3(1-x)(1+x+x^2)}\right]$

$\lim_{x\to 1}\left[\frac{-(x^2+x-2)}{3(x-1)(1+x+x^2)}\right]$

$\frac{3}{3(1+1+1)}$

$\frac{1}{3}$

D) $\lim_{x\to 0}\frac{\tan 3x}{\tan 2x}$

$\lim_{x\to 0}\frac{\sin 3x}{3x}\cdot\frac{\cos 2x}{\cos 3x}\cdot\frac{2x}{\sin 2x}\cdot\frac{3}{2}$

$1\cdot 1\cdot 1\cdot\frac{3}{2}=\frac{3}{2}$

E) $\lim_{x\to 0}\left(1+\frac{1-\cos x}{x}\right)^{\frac{\sin x}{x}}$

$(1+0)^1$

1

F) $\lim_{x\to 2}\frac{\sin(5x-10)}{3x-6}$

$\lim_{x\to 2}\frac{\sin(5x-10)}{\frac{3}{5}\cdot 5(x-2)}=\frac{5}{3}$

G) $\lim_{x\to 0}\frac{\pi\sec\left(x+\frac{\pi}{3}\right)-\pi\sec\frac{\pi}{3}}{x}$

Def of Deriv of $\pi\sec x$ at $x=\frac{\pi}{3}$

$f'=\pi\sec x\tan x\Big|_{x=\frac{\pi}{3}}$

$\pi\cdot 2\cdot\sqrt{3}$

$2\pi\sqrt{3}$

V let $h=f(x)-g(x)$

$h(x)=x^4-5x^2-(2x^3-1x+6)$

$=x^4-2x^3-5x^2+4x-6$

1. $-2\quad -5\quad 4\quad -6$

3] $1\quad 1\quad -2\quad -2\quad -12$

4] $1\quad 2\quad 3\quad 16\quad 58$

Since h is continuous in $[3,4]$ and since $h(a)$ Lo and $h(b)>0$ IVT Guarantees $\exists\, h(x)=0$.

Extra.

$900\cdot\frac{1}{60}$ $210\cdot\frac{1}{60}$

150 m/min 350 m/min.

$150+350=500$ units in one minute. They are 500 m apart.

105

100	2
90	3
80	9
70	6
40	1
59	3

45

1919
24

80.0

Great exam!

10|12|98

100	0
90	1
80	8
70	4
60	2
59	0

1202
15

80.1

Great Exam!

A.P. Calculus Name Col ch1-4

Copy original problem. Per _____ Date _____

Convince *me* that **you** understand the concept*!*

No calculators.

Chapters 1 → 4 Exam

I The definition of continuity is actually a collection of three "tests". For each of the following, draw a sketch which illustrates the situation when only the "item" listed fails. (15 pts/tot)

a) Item 1 b) Item 2 c) Item 3

II Given $\frac{1+\sin x}{1-2\sin x}=\frac{3}{4}$. Find $\cos 2x$ (15 pts)

III Given $f(x)=\begin{cases} x^2 & ; \ x\le 1 \\ 2x-1 & ; \ x>1 \end{cases}$ (10 pts ea)

a) ***Prove*** $f(x)$ is continuous at $x=1$.

b) ***Prove*** $f'(1)$ does or does not exist.

IV Determine the following limits: (5 pts ea)

a) $\lim_{x\to 2}\frac{\sqrt{x^2+5}-3}{x-2}$

b) $\lim_{x\to 0}\sqrt{1+\frac{1}{x}}-\sqrt{\frac{1}{x}}$

c) $\lim_{x\to 1}\left[\frac{1}{1-x^3}-\frac{1}{3(1-x)}\right]$

d) $\lim_{x\to 0}\left(\frac{\tan 3x}{\tan 2x}\right)$

e) $\lim_{x\to 0}\left(1+\frac{1-\cos x}{x}\right)^{\frac{\sin x}{x}}$

f) $\lim_{x\to 2}\frac{\sin(5x-10)}{3x-6}$

g) $\lim_{x\to 0}\left(\frac{\pi\sec\left(x+\frac{\pi}{3}\right)-\pi\sec\frac{\pi}{3}}{x}\right)$

V Use the Intermediate Value Theorem to show that the graphs of $f(x)=x^4-5x^2$ and $g(x)=2x^3-4x+6$ intersect between $x=3$ and $x=4$. (15 pts)

Extra Credit ---------- 5 pts ----------

Two missiles speed directly toward each other, one at 9,000 MPH and one at 21,000 MPH. If they start at 1,317 miles apart, how far apart are they one minute before they collide?

2001

74 Kent Boes
72 Jimmy Alonso
72 Fatima Iqbal
60 Agatha McCallister
60 Lilly Chow
60 Justin Gilmore.

Pretty sad Showing.
The top two were too cool
to study and got toasted
Iqbal Didn't better. effort
but made many careless errors.
Bottom 3 don't belong
here.

10/18/2000

Deanna Strother 94
Travis Treadwell 93
Steven Schlabach 90
Jon Valencia 89
Melanie Boudoin 88
Esmeralda Gonzales 84
Jeff Mohr 81
Daryl Gaskin 77
Lauren Hylton 77
Elaine Man 69

10/23/2002

Dani Cepernich 95
Allison Barrie 92
Lauren Davis 92
Nick Chambers 91
Eamon Golan 88
Craig Abernathy 83
Thomas Olson 79
Rebecca Branco 78
Joel Hanks 75
Andrea Thomas 74
Kyle Sheeley 71
Rachel Heinz 70

Elizabeth Lampler 65
Lauren Takeda 42

10/24/96

Ryan Swenston 87
Hwa-Shang Lu 65
Chris Bunday 62
Ben Dawson 56
Chrystal Brody 58

I need one change as indicated.
This should have been very easy.
This shows that they generally Don't understand what they're doing.
The CAN do but Don't Know why!

98

Matt Mandeville 84
Amy Phillippi 84
Mike McCulley 71
Lee Hong 68
Matt Irwin 66
Daniel Wesp 65

I 1) if f is continuous on [a,b] and m is a number between f(a) and f(b) then ∃ x ∋ f(x) = m.
that is, a continuous fcn takes on all values between f(a) and f(b) and there is a Domain value x for each Range value.

2) 2 fcns intersect when the difference in their range values at that point would be zero. Define a third fcn which is the difference between the first two. If you can show the 3rd fcn is neg and then pos in between (you know it is continuous, then you can use IVT to conclude that f(x) = 0 so the two original fcns had a common intersection

3) Let f(x) = $ax^n + bx^{n-1} + \dots + cx + e$ where n is odd, f is cont since it is a polynomial fcn,
$\lim_{x\to-\infty} f(x) < 0$ and $\lim_{x\to+\infty} f(x) > 0$ ∴ ∃ x ∈ (−∞, ∞) such that f(x) = 0 that x is the value of f(x) by IVT,

II $f(x) = \begin{cases} 3x-2 & \text{if } x<1 \\ x^3 & \text{if } x\geq 1 \end{cases}$

① f(1) = 1

4) $\lim_{x\to1^-} f(x) = \lim_{x\to1^-} 3x-2 = 1$

$\lim_{x\to1^+} f(x) = \lim_{x\to1^+} x^3 = 1$

$\lim_{x\to1} f(x) = 1$ ② ← = ③ ∴ f is cont at x = 1

5) $\lim_{x\to1^-} \frac{f(x)-f(a)}{x-1}$

$\lim_{x\to1^-} \frac{3x-2-1}{x-1}$ | $\lim_{x\to1^+} \frac{x^3-1}{x-1}$

$\lim_{x\to1^-} \frac{3(x-1)}{x-1}$ | $\lim_{x\to1^+} \frac{(x-1)(x^2+x+1)}{x-1}$

3 = 3

∴ f'(1) = 3.

c) $f'(x) = \begin{cases} 3 & \text{if } x<1 \\ 3x^2 & \text{if } x\geq 1 \end{cases}$

The only point in question is at x = 1

Proof: f'(1) = 3 ①

$\lim_{x\to1^-} f'(x) = 3$

$\lim_{x\to1^+} f'(x) = \lim_{x\to1^+} 3x^2 = 3$ = ③ ∴ f' is cont at x = 1

∴ $\lim_{x\to1} f'(x) = 3$ ② ←

III $f(x) = \begin{cases} px+q & \text{if } x < \frac{1}{2} \\ 3x^2 & \text{if } x \geq \frac{1}{2} \end{cases}$

A) $\lim_{x \to \frac{1}{2}^-} f(x)$

$\lim_{x \to \frac{1}{2}^-} px+q = \frac{1}{2}p+q$

B) $\lim_{x \to \frac{1}{2}^+} f(x)$

$\lim_{x \to \frac{1}{2}^+} 3x^2 = \frac{3}{4}$

C) $\frac{1}{2}p+q = \frac{3}{4}$

$q = \frac{3}{4} - \frac{1}{2}p$

D) $\lim_{x \to \frac{1}{2}} \frac{f(x)-f(a)}{x-a}$

$\lim_{x \to \frac{1}{2}^-} \frac{px+q-\frac{3}{4}}{x-\frac{1}{2}}$

$\lim_{x \to \frac{1}{2}^-} \frac{px+\frac{3}{4}-\frac{1}{2}p-\frac{3}{4}}{x-\frac{1}{2}}$

$\lim_{x \to \frac{1}{2}^-} \frac{p(x-\frac{1}{2})}{x-\frac{1}{2}} = p$

$\lim_{x \to \frac{1}{2}^+} \frac{3x^2-\frac{3}{4}}{x-\frac{1}{2}}$

$\lim_{x \to \frac{1}{2}^+} \frac{3(x-\frac{1}{2})(x+\frac{1}{2})}{(x-\frac{1}{2})}$

$3(\frac{1}{2}+\frac{1}{2})$

3

Since (L) = (R) $3 = p$

$\therefore q = \frac{3}{4} - \frac{1}{2}3$

$q = -\frac{3}{4}$

f'rd $f'(\frac{1}{2})$ exists and $= 3$

IV A) $\lim_{x \to \frac{1}{3}} \frac{9x^2-1}{3x+1}$

$\lim_{x \to \frac{1}{3}} \frac{(3x-1)(3x+1)}{(3x+1)}$

$-1-1 = -2$

B) $\lim_{x \to -\infty} \frac{x^2+4}{x^4-1}$ (÷ $\frac{1}{x^4}$)

$\lim_{x \to -\infty} \frac{\frac{1}{x^2}+\frac{4}{x^4}}{1+\frac{1}{x^4}}$

$\frac{0+0}{1-0} = 0$

C) $\lim_{x \to 0} \frac{2\sin 3x}{5x} \cdot \frac{3}{3}$

$\lim_{x \to 0} \frac{6\sin 3x}{5 \cdot 3x}$

$\frac{6}{5}$

E) $\lim_{x \to \frac{\pi}{4}} \frac{2x-\frac{\pi}{2}}{\cos 2x}$ Let $A = x - \frac{\pi}{4}$, $x = A + \frac{\pi}{4}$

$\lim_{A \to 0} \frac{2A}{\cos 2(A+\frac{\pi}{4})}$

$\lim_{A \to 0} \frac{2A}{\cos(2A+\frac{\pi}{2})}$

$\lim_{A \to 0} \frac{2A}{\cos 2A \cos\frac{\pi}{2} - \sin 2A \sin\frac{\pi}{2}}$

$\lim_{A \to 0} \frac{2A}{0-\sin 2A}$

-1

F) $\lim_{x \to 0} \frac{\tan 2x}{x}$

$\lim_{x \to 0} 2\frac{\sin 2x}{2x\cos 2x}$ → $\lim_{x \to 0} \frac{2}{\cos 2x} \cdot \frac{\sin 2x}{x}$

$\frac{2}{1} \cdot 1$

2

V) $\lim_{x \to -\infty} \frac{\sqrt{x^2+4x+4}}{2x+3}$

$\lim_{x \to -\infty} \frac{-x\sqrt{1+\frac{4}{x}+\frac{4}{x^2}}}{x(2-\frac{3}{x})}$

$-\frac{1}{2}$

VI) A) $\lim_{x \to 2} \frac{\sqrt{x+4}-4}{x-2} \cdot \frac{\sqrt{x}+4}{\sqrt{x}+4}$

$\lim_{x \to 2} \frac{x-2}{x-2(\sqrt{x}+4)}$

$\lim_{x \to 2} \frac{1}{\sqrt{x}+4}+4$

$\frac{1}{4+4} = \frac{1}{8}$

A.P. Calculus
Copy original problem.
Convince *me* that **you** understand the concept!
No calculators, of course.

Name _____

Per _____ Date _____

Make Up Exam #2 Quarter #1 Chapters 1 – 4

I Intermediate Value Theorem (20 pts tot)

A) Without selecting specific functions, explain what the Intermediate Value Theorem is and explain how it might be used to show that two functions have a common intersection point. I am *not* looking for specific examples but you must be explicit in your description.

B) Using the IVT, prove that a polynomial of odd degree must have at least one real root.

II Given: $f(x)=\begin{cases} 3x-2 & \text{if } x<1 \\ x^3 & \text{if } x\geq 1 \end{cases}$ (25 pts tot)

A) Prove f is continuous at $x = 1$.

B) Prove $f'(1)$ does exist.

C) Prove $f'(x)$ is continuous.

III Given: $f(x)=\begin{cases} Px+Q & \text{if } x<\frac{1}{2} \\ 3x^2 & \text{if } x\geq\frac{1}{2} \end{cases}$ is a continuous function. (20 pts tot)

A) Find $\lim_{x\to\frac{1}{2}^-} f(x)$.

B) Find $\lim_{x\to\frac{1}{2}^+} f(x)$.

C) Since f is continuous, what can be concluded from steps A and B? (ie. solve for Q)

D) Determine P and Q so that $f'\left(\frac{1}{2}\right)$ exists.

IV Determine the following limits, if possible. Briefly explain your reasoning. (5 pts ea)

A) $\lim_{x\to -\frac{1}{3}} \frac{9x^2-1}{3x+1}$ B) $\lim_{x\to 2} \frac{\sqrt{x+14}-4}{x-2}$ C) $\lim_{x\to -\infty} \frac{x^2+4}{x^4-1}$ D) $\lim_{x\to -\infty} \frac{\sqrt{x^2+4x+4}}{2x+3}$

E) $\lim_{x\to 0} \frac{2\sin 3x}{5x}$ F) $\lim_{x\to \frac{\pi}{4}} \frac{2x-\frac{\pi}{2}}{\cos 2x}$ G) $\lim_{x\to 0} \frac{\tan 2x}{x}$

I $f(x) = \frac{1}{1-x}$ $x \neq 1$

A) $\lim_{h\to 0} \frac{1}{h}\left(\frac{1}{1-x-h} - \frac{1}{1-x}\right)$

$\lim_{h\to 0} \frac{1}{h}\left[\frac{1-x-(1-x-h)}{(1-x-h)(1-x)}\right]$

$\lim_{x\to 0} \frac{1}{h}\left[\frac{1-x-1+x+h}{(1-x-h)(1-x)}\right]$

$\frac{1}{(1-x)^2} = f'(x)$

B) $f(3) = \frac{1}{1-3} = -\frac{1}{2}$ $f'(3) = \frac{1}{(1-3)^2} = \frac{1}{4}$ skipped this_5

$y + \frac{1}{2} = \frac{1}{4}(x-3)$

$= \frac{1}{4}x - \frac{3}{4} - \frac{2}{4}$

$y = \frac{1}{4}x - \frac{5}{4}$ set $= 0$

$x = 5$.

the x intercept is $(5, 0)$ 3pts

C)

$x + 4y - 2 = 0$

$4y = -x + 2$

$y = -\frac{1}{4}x + \frac{1}{2}$

$m = 4 \overset{\text{set}}{=} \frac{1}{(1-x)^2}$

$4x^2 - 8x + 4 = 1$

$4x^2 - 8x + 3 = 0$

$(2x-1)(2x-3) = 0$

$x \in \{\frac{1}{2}, \frac{3}{2}\}$.

$f(\frac{1}{2}) = \frac{1}{1-\frac{1}{2}} = 2$ $f(\frac{3}{2}) = \frac{1}{1-\frac{3}{2}} = -2$

$(\frac{1}{2}, 2)$ $m = 4$ $(\frac{3}{2}, -2)$ $m = 4$

$y - 2 = 4(x - \frac{1}{2})$ $y + 2 = 4(x - \frac{3}{2})$

$y - 2 = 4x - 2$ $y + 2 = 4x - 6$

$y = 4x$ $y = 4x - 8$

4 pts 4 pts

II

A) Given an equation, you would determine a fcts which had the same zeros as the equation had roots. You would find an interval $[a, b]$ such that $f(x)$ would change signs. Then by the IVT, there would be a Σ in $[a, b]$ such that $f(x)$ would equal zero. Since f had the same zeros as the equations had roots, the given equation would have at least one root. continuous

B) $\lim_{x\to a} f(x) \overset{②}{=} f(a)$ ① is the def of continuity (which has 3 parts)

$\lim_{x\to a} f(x) = L$ means $\lim_{x\to a^-} f(x) = L_L$; $\lim_{x\to a^+} f(x) = L_R$ and

4 pts. $L_L = L_R \overset{\text{then}}{=} L$.

III ↑ $f(x) = \begin{cases} \frac{x^4-1}{x-1} & x \neq 1 \\ K & x = 1 \end{cases}$

$\frac{x^4-1}{x-1} = \frac{(x^2-1)(x^2+1)}{x-1}$

$= (x+1)(x^2+1)$

Def of Lim $\lim_{x\to 1} (x+1)(x^2+1) = 4 \therefore K = 4$ to be continuous. Only then

Dr $x \to 1$

$\therefore$ $\lim_{x\to 1} \frac{(x+1)(x^2+1) - 4}{x-1}$ −8

$\frac{x^2+x^3+x+1-4}{x-1}$

	1	1	1	-3
1	1	2	3	0

x^2+2x+3

$\lim_{x\to 1} x^2+2x+3 = 6$

Since this limit is the def of the der. the der exists.

D) $\lim_{x\to\pi} \frac{|\sin x|}{x-\pi}$ let $x-\pi = A$

$\lim_{A\to 0} \frac{|\sin(A+\pi)|}{A}$

$\lim_{A\to 0} \frac{|\sin A \cos\pi + \sin\pi \cos A|}{A}$ ($\sin\pi = 0$)

$\lim_{A\to 0} \frac{|-\sin A|}{A}$

$\lim_{A\to 0^-} \frac{\sin A}{A} = 1$

$\lim_{A\to 0^+} \frac{-\sin A}{A} = -1$

DNE (L) ≠ (R)

B) $f(x) = \begin{cases} x+1 & x > 0 \\ K & x = 0 \\ x-1 & x < 0 \end{cases}$

$\lim_{x\to 0^-} x-1 = -1$ def & derivative 8

$\lim_{x\to 0^+} x+1 = 1$

(L) ≠ (R) ∴ f not cont at $x=0$. Since f not cont, f' cannot exist.

IV

A) $\lim_{x\to 1^+} \frac{\sin\sqrt{1-x}}{\sqrt{1-x}}$ DNE CANNot approach

B) $\lim_{x\to 0} \frac{\sin x \cos x - 2\sin x}{x}$

$\lim_{x\to 0} \frac{\sin x}{x}[\cos x - 2]$

$1[1-2] = -1$

C) $\lim_{x\to 1^+} (2-x)^{\frac{1}{x-1}}$

$\lim_{x\to 1^+} \left[(1+(1-x))^{\frac{-1}{1-x}}\right]^{-1} = e^{-1} = \frac{1}{e}$

E) $\lim_{x\to 3^-} \frac{|x+3|}{\sqrt{x}} = \frac{6}{\sqrt{3}} = 2\sqrt{3}$ Neg = −1

Extra:

$f(x) = \sqrt{x^2+9}$ Sub $x - \sqrt{x}$

$f(g(x)) = \sqrt{(g(x))^2+9}$

$(g(x))^2 = x^2 - 2x\sqrt{x} + x + 9$

$g(x) = \pm\sqrt{x^2 + x - 2x\sqrt{x} + 9}$

	4	5	4
100	0	1	1
90	0	1	1
80	4	3	7
70	3	7	10
60	7	3	10
54	3	3	6

102

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$\frac{1150}{17}$ $\frac{1319}{18}$ $\frac{2469}{35}$

67.6 73.3 70.5

A.P. Calculus

Copy original problem.

Convince *me* that **you** understand the concept!

Each section 25 pts.

No calculators, of course.

Name Calc 2nd Exam

Per _____ Date 2002

Exam # 2

I Given $f(x) = \dfrac{1}{1-x}$; $x \neq 1$

A) Using the definition of the derivative, find $f'(x)$.

B) Determine the x-intercept of the tangent line to $f(x)$ at $x = 3$

C) Find the equation(s) of all tangent lines to $f(x)$ which are perpendicular to: $x + 4y - 2 = 0$.

II Explain 'ums

A) **Without using examples**, explain how you could use the Intermediate Value Theorem to prove that an equation had a solution.

B) Explain everything there is to know about: $\lim\limits_{x \to a} f(x) = f(a)$.

III Given $f(x)$. If possible, determine k so that $f(x)$ will be differentiable.
Prove that your value of k works.

A) $$f(x) = \begin{cases} \dfrac{x^4 - 1}{x - 1} & x \neq 1 \\ k & x = 1 \end{cases}$$

B) $$f(x) = \begin{cases} x + 1 & x > 0 \\ k & x = 0 \\ x - 1 & x < 0 \end{cases}$$

IV Determine the following limits, if possible.

A) $\lim\limits_{x \to 1^+} \dfrac{\sin\sqrt{1-x}}{\sqrt{1-x}}$

B) $\lim\limits_{x \to 0} \dfrac{\sin x \cos x - 2\sin x}{x}$

C) $\lim\limits_{x \to 1} (2 - x)^{\frac{1}{x-1}}$

D) $\lim\limits_{x \to \pi} \dfrac{|\sin x|}{x - \pi}$

E) $\lim\limits_{x \to 3^-} \dfrac{|x + 3|}{\sqrt{x}}$

Extra Credit ---------- 5 pts ----------

Let $f(x) = \sqrt{x^2 + 9}$. Determine two different functions $g(x)$ so that $f(g(x)) = x - \sqrt{x}$.

ADVANCED PLACEMENT CALCULUS AB
SECOND EXAM ——FIRST QUARTER

You *must be experts* on: **1. Definition of Continuity** **2. Definition of the derivative (both)** **3. Intermediate Value Thm** **4. and the *proper use* of the above 3 items** ***before this second exam of the quarter !***			EXAM Ch 1 --> Ch 4.4 #1 R/C Pg 77 - 84 #2 Pg 84 1 - 6, 11, 13, 29 9/26	#3 Pg 85 - 86 17, 19 - 28 9/27
#4 Pg 89 1 - 10 **Important lecture** 9/30	#5 Pg 89 11 - 17, 19, 20 #6 R/C pg 90 - 95 10/1	#7 Pg 95 - 96 1 - 24, 28, 30 10/2	#8 GQ EXP 1 - 5 GQ Trg 1, 2, 3 Pg 98 - 99 GQ Ch 4 1 - 8 Pg 99 RE 2 - 5, 7, 8 10/3	#9, #10 Pg 100 Ch 4 REV 11 - 15 Pg 100 - 102 RE Ch 1 - 4 1 - 8, 10,11, 10/4
#11 Pg 100 - 102 RE Ch 1 - 4 13 - 19, 21 – 27, 31 - 38 10/ 7	begin "Continuous Enjoyment 2nd Edition" **Each** problem requires comments. brief statement describing methods #12 #13 pblms 1 - 27 10/8	#14, #15 "Continuous Enjoyment 2nd Edition" problems 28 - 53 10/ 9	"Continuous Enjoyment 2nd Edition" finish 10/ 10	REVIEW 10/ 11
EXAM **CHAPTERS 1 —> 4** 10/ 14		**2002**		

I $f(x) = \frac{1}{1-x}$, $x \neq 1$

A) $\lim_{h\to 0} \left(\frac{1}{1-(x+h)} - \frac{1}{1-x}\right)\frac{1}{h}$

$\lim_{h\to 0} \frac{1}{h}\left[\frac{1-x-1+(x+h)}{(1-(x+h))(1-x)}\right]$

$\lim_{h\to 0} \frac{1-x-1+x+h}{h(1-x-h)(1-x)}$

$\frac{1}{(1-x)^2}$

II $f(x) = \begin{cases} \frac{x^4-1}{x-1} & x \neq 1 \\ k & x = 1 \end{cases}$

A) $\lim_{x\to 1} f(x) =$

$\lim_{x\to 1} \frac{(x-1)(x+1)(x^2+1)}{(x-1)}$

$2 \cdot 2 = 4$

K must be 4 so $\lim_{x\to a} f(x) = f(a)$

B) $\lim_{x\to 1} \frac{\frac{x^4-1}{x-1} - 4}{x-1}$

$\lim_{x\to 1} \frac{x^4-1-4x+4}{(x-1)^2}$

$\lim_{x\to 1} \frac{x^4-4x+3}{(x-1)^2}$

The limit exists... There exists at $x=1$ which is the only questionable spot.

	1	0	0	-4	3
1	1	1	1	-3	0
1	1	2	3	0	

$\lim_{x\to 1} x^2+2x+3 = 6$

The limit exists

B) $(3, f(3))$ $\quad y - y_1 = m(x - x_1)$

$f(3) = \frac{1}{1-3} = \frac{-1}{2}$ $\quad y - (-\frac{1}{2}) = \frac{1}{4}(x-3)$

$m = \frac{1}{(1-x)^2}\Big| = \frac{1}{(-2)^2} = \frac{1}{4}$ $\quad y + \frac{1}{2} = \frac{1}{4}x - \frac{3}{4}$

$y = \frac{1}{4}x - \frac{5}{4}$

$x = 3$ 3 PTS

C) Both $\quad x + 4y - 2 = 0$

$y = -\frac{1}{4}x + \frac{1}{2}$ $\quad m = -\frac{1}{4}$

So we need $m = +4$

$\frac{1}{(1-x)^2} \overset{\text{set}}{=} 4$

$1 = 4 - 8x + 4x^2$

$4x^2 - 8x + 3 = 0$

$(2x-3)(2x-1) = 0$

$x = \{\frac{3}{2}, \frac{1}{2}\}$.

$f(\frac{3}{2}) = \frac{1}{1-\frac{3}{2}} = \frac{1}{\frac{-1}{2}} = -2$

$y + 2 = 4(x - \frac{3}{2})$

$y + 2 = 4x - 6$

$y = 4x - 8$

$f(\frac{1}{2}) = \frac{1}{1-\frac{1}{2}} = 2$

$y - 2 = 4(x - \frac{1}{2})$

$y = 4x$

III $f(x) = \begin{cases} Px+Q & x < \frac{1}{2} \\ 3x^2 & x \geq \frac{1}{2} \end{cases}$ f is continuous

a) $\lim_{x \to \frac{1}{2}^-} Px+Q = \frac{1}{2}P + Q$

b) $\lim_{x \to \frac{1}{2}^+} 3x^2 = 3\left(\frac{1}{2}\right)^2 = \frac{3}{4}$

c) $\frac{1}{2}P + Q \overset{\text{Set}}{=} \frac{3}{4}$

$Q = \frac{3}{4} - \frac{1}{2}P$

d) $\lim_{x \to \frac{1}{2}^-} \frac{Px+Q-\frac{3}{4}}{x-\frac{1}{2}}$

$\lim_{x \to \frac{1}{2}^-} \frac{4Px+4Q-3}{4x-2}$

$\lim_{x \to \frac{1}{2}^-} \frac{4Px+4Q-3}{2(2x-1)}$

$\lim_{x \to \frac{1}{2}^-} \frac{4Px+4\left(\frac{3}{4}-\frac{1}{2}P\right)-3}{2(2x-1)}$

$\lim_{x \to \frac{1}{2}^-} \frac{4Px+3-2P-3}{2(2x-1)}$

$\lim_{x \to \frac{1}{2}^-} \frac{4Px-2P}{2(2x-1)}$

$\lim_{x \to \frac{1}{2}^-} \frac{2P(2x-1)}{2(2x-1)} = P$

$\lim_{x \to \frac{1}{2}^+} \frac{3x^2-\frac{3}{4}}{x-\frac{1}{2}}$

$\lim_{x \to \frac{1}{2}^+} \frac{12x^2-3}{4} \Big/ 2(2x-1)$

$\lim_{x \to \frac{1}{2}^+} \frac{3(4x^2-1)}{2(2x-1)}$

$\lim_{x \to \frac{1}{2}^+} \frac{3(2x-1)(2x+1)}{2(2x-1)}$

$\lim_{x \to \frac{1}{2}^+} \frac{3(2x+1)}{2}$

$3 \cdot \frac{2}{2} = 3$

So $P = 3$.

But $Q = \frac{3}{4} - \frac{1}{2}P$

$Q = \frac{3}{4} - \frac{3}{2}$

$Q = -\frac{3}{4}$

So $f'\left(\frac{1}{2}\right)$ exists AND is $= 6$ 3.

IV a) $\lim_{x \to 0} \frac{1-\cos^2 x + \sin^2 3x}{x^2}$

$\lim_{x \to 0} \frac{1-\cos^2 x}{x^2} + \frac{9\sin^2 3x}{9x^2}$

$\frac{\sin^2 x}{x^2} + 9 = 10$

b) $\lim_{x \to 2} \frac{\sqrt{2x^2+8}-4}{x-2} \cdot \frac{\sqrt{\quad}+4}{\sqrt{\quad}+4}$

$\lim_{x \to 2} \frac{2x^2+8-16}{(x-2)(\sqrt{\quad}+4)}$

$\lim_{x \to 2} \frac{2(x^2-4)}{x-2(\sqrt{\quad}+4)}$

$\lim_{x \to 2} \frac{2(x+2)}{\sqrt{2x^2+8}+4}$ $\frac{2 \cdot 4}{\sqrt{16}+4} = \frac{8}{8} = 1$

c) $\lim_{x \to 0} \frac{x+|x|}{\sqrt{x^2}}$ $\sqrt{x^2} = |x|$

$\lim_{x \to 0^-} \frac{x-x}{-x}$ $\lim_{x \to 0^+} \frac{2x}{x}$

$\lim_{x \to 0^-} \frac{0}{-x}$ $\lim_{x \to 0^+} 2 = 2$

$\lim_{x \to 0^-} 0 = 0$ $\neq$

(b) $\neq$ R DNE.

d) $\lim_{x \to 0} (1+\sin x)^{\frac{2}{x}}$

$\lim_{x \to 0} \left[(1+\sin x)^{\frac{\sin x}{x}}\right]^{\frac{2}{\sin x}}$

$\lim_{x \to 0} \left[(1+\sin x)^{\frac{1}{\sin x}}\right]^{2 \cdot \frac{\sin x}{x}}$

e^2 $\leftarrow$ $2 \cdot 1$

Replace I with a
Problem (or problems)
dealing with IVT.

change III d

~~Using 1st Def of Der~~ ~~Do~~ Set up
Consider ~~The left hand~~ portions of the
~~Def of~~
~~Use Def of Der~~
use 1st Def of Dr. Notice L hand limit
appears different than R hand limit.
Set your answer to III c into for
Q and simplify.
Since (L) $\neq$ (R) what is your conclusion?

III e Solve for Q.

Jeff Bell 94
Jerod Bonian 93
ANNMarie Conti 80
Natalie Dumont 80
Pat Wibbeler 77
ANDY D'acquisto 75
Jeff Bross 70
Trevor Wilson 70
Pam Ryon 65
Lavona Hamilton 65
Carey Dmytrie 61
Kristy MATTson 56
Jennifer Noonan 53
Jay Wagner 51
Cyndi Thompson 41
Kathy Libal 31

Advanced Placement Calculus

Name 92

Copy original problem.

Per _____ Date _____

Convince *me* that **you** understand the concept!

Chapters 1– 4 Make-Up Exam

I Given $f(x)=\dfrac{1}{1-x}$, $x\neq 1$. (10 pts ea)

a) Using the definition of the derivative, find $f'(x)$.

b) Determine the equation of the tangent line to $f(x)$ at $(3, f(3))$.

c) Find the equation of both tangent lines to $f(x)$ that are perpendicular to : $x+4y-2=0$.

II Given $f(x)=\begin{cases}\dfrac{x^4-1}{x-1} & x\neq 1\\ k & x=1\end{cases}$ (total 25 pts)

a) Determine k so $f(x)$ is continuous for all x.

b) Prove $f'(x)$ does or does not exist at all questionable values of x.

III Given $f(x)=\begin{cases}Px+Q & x<\frac{1}{2}\\ 3x^2 & x\geq\frac{1}{2}\end{cases}$; $f(x)$ is continuous. (total 25 pts)

a) Find $\lim\limits_{x\uparrow\frac{1}{2}} f(x)$.

b) Find $\lim\limits_{x\downarrow\frac{1}{2}} f(x)$.

c) Since $f(x)$ is continuous, what can be concluded from steps a and b above? (i.e. Solve for Q in terms of P.)

d) Determine P and Q so that $f'\left(\frac{1}{2}\right)$ exists. (You will need to substitute your answer to part c into your left-hand limit.)

IV Determine the following limits, if possible. ***Briefly*** explain your reasoning. (5 pts ea)

a) $\lim\limits_{x\to 0}\dfrac{1-\cos^2 x+\sin^2 3x}{x^2}$

b) $\lim\limits_{x\to 2}\dfrac{\sqrt{2x^2+8}-4}{x-2}$

c) $\lim\limits_{x\to 0}\dfrac{x+|x|}{\sqrt{x^2}}$

d) $\lim\limits_{x\to 0}(1+\sin x)^{\frac{2}{x}}$

I

$$f(x)=\begin{cases}|x+3| & x\le -2\\ x^2+1 & -2<x<1\\ 16\sqrt{x+3}-30 & x\ge 1\end{cases}$$

A) at $x=-2$ $\quad f(-2)=1$

$\lim_{x\to -2^-}|x+3|=1$ $\quad \lim_{x\to -2^+}x^2+1=5$

Limit DNE ↑ ≠ ↑

fctn not cont at $x=-2$

B) at $x=-3$

$\lim_{x\to -3}\frac{|x+3|-0}{x+3}$

$\lim_{x\to -3^-}\frac{-(x+3)}{x+3}=-1$

$\lim_{x\to -3^+}\frac{x+3}{x+3}=1$

(L) ≠ (R) ∴ limit DNE and Limit is Defined So Der DNE

at $x=-2$ Der DNE since f is not (-1) continuous

at $x=1$

$\lim_{x\to 1^-}\frac{x^3+1-2}{x-1}$

$\lim_{x\to 1^-}\frac{x^2-1}{x-1}$

$\lim_{x\to 1^-}x+1=2$

(L) ≠ (R) $2\neq 4$ ∴ limit DNE since limit is Def of Der. Der DNE.

at $x=1$ $\quad f(1)=2$

$\lim_{x\to 1^-}x^3+1=2$ $\quad \lim_{x\to 1^+}16\sqrt{x+3}-30=2$

$\lim_{x\to 1}f(x)=2=f(1)$

f is cont at $x=1$

$\lim_{x\to 1^+}\frac{16\sqrt{x+3}-30-2}{x-1}$,

$\lim_{x\to 1^+}\frac{16\sqrt{x+3}-32}{x-1}$

$\lim_{x\to 1^+}16\frac{\sqrt{x+3}-2}{x-1}\frac{\sqrt{}+2}{\sqrt{}+2}$ ⑤

$\lim_{x\to 1^+}\frac{16(x+3-4)}{(x-1)(\sqrt{x+3}+2)}$

limit is $\frac{16}{4}=4$

Der DNE.

II

① Define a 3rd fctn $h(x)=f(x)-g(x)$.

② Determine that h(x) is continuous for what interval h is continuous.

③ Determine endpts a, b of the interval when h(a) has the opposite sign of h(b).

④ The IVT guarantees a $\bar{x}$ such that $h(\bar{x})=0$ which is between h(a) and h(b).

If $h(\bar{x})=0$ then $f(\bar{x})=g(\bar{x})$ which was the requirement.

III

A) $\lim_{x\to 0} \frac{\cos(x+\frac{\pi}{2})}{x}$

$\lim_{x\to 0} \frac{\cos x \cos\frac{\pi}{2} - \sin x \sin\frac{\pi}{2}}{x}$ ($\cos\frac{\pi}{2} = 0$)

$\lim_{x\to 0} \frac{-\sin x}{x} = -1$

b) $\lim_{x\to\infty} \frac{3x^2-4}{2-7x-x^2} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}}$

$\lim_{x\to\infty} \frac{3-\frac{4}{x^2}}{\frac{2}{x^2}-\frac{7}{x}-1} = -3$

c) $\lim_{x\to\infty} \frac{\sqrt{x}-4}{4-3\sqrt{x}} \cdot \frac{\frac{1}{\sqrt{x}}}{\frac{1}{\sqrt{x}}}$

$\lim_{x\to\infty} \frac{1-\frac{4}{\sqrt{x}}}{\frac{4}{\sqrt{x}}-3} = -\frac{1}{3}$

d) $\lim_{x\to 0} \frac{\sin(x+\frac{\pi}{2})-1}{x}$

$\lim_{x\to 0} \frac{\sin x \cos\frac{\pi}{2} + \sin\frac{\pi}{2}\cos x - 1}{x}$ ($\sin x \cos\frac{\pi}{2} = 0$, $\sin\frac{\pi}{2} = 1$)

$\lim_{x\to 0} \frac{\cos x - 1}{x} = 0$

e) $\lim_{x\to 3^-} \frac{|x+3|}{\sqrt{x}}$

$\frac{6}{\sqrt{3}}$

$\frac{6\sqrt{3}}{3}$

$2\sqrt{3}$

f) $\lim_{x\to\infty} x \tan\frac{\pi}{x}$

$\lim_{x\to\infty} \frac{\sin\frac{\pi}{x}}{\frac{1}{x}} \cdot \frac{\pi}{\cos\frac{\pi}{x}}$

$1 \cdot \frac{\pi}{1} = \pi$

G) $\lim_{x\to\infty} \left(\left(\frac{7}{5}\right)^x + \frac{3x^3+4}{7-4x^3}\right)$

$\left(\frac{5}{7}\right)^{\text{large}} + \frac{3+\frac{4}{x^3}}{\frac{7}{x^3}-4}$

$0 \quad -\frac{3}{4}$

E) T-a.

$\frac{2}{3}$ men married to $\frac{3}{5}$ women

LCD$(2,3) = 6$ (must be the same number)

6 men are married, 6 women are married

$\frac{6}{9}$ men $\rightarrow \frac{6}{10}$ women. 9+10 = 19 units. of 19 units, 12 are married

There is a total of 19 units (9+10) of people. 6 of the men units and 6 of the women units are married. ie a total of 12 married units leaving 7 unmarried units. Then $\frac{7}{19}$ are single.

92

Many LAZY Students.
Quality of HW very low.
Many have the "swollen head"
of being the 6TH ranked Senior etc.

INsist on Seniors from Bell to Bell!
They're slugs to get started each day.
It certainly looks like almost everybody
needs their wisdom molded up.

100↑	0	96
90↑	1	
80↑	3	
70↑	8	
60↑	7	38
59↓	5	

$\frac{1608}{24} = 67.0$

94

100	0
90	0
80	5
70	10
60	1
59	0

87

$\frac{1228}{10} =$

76.8

60

Poor ones to II = -10
must get -8.
NO Probs in I -15 -> -20
DIDN'T have 3 pts in I_B -10
good results. HW Grades are
very high so This brings Things
back in Line.

Advanced Placement Calculus

Name 92/94

Copy original problem. Per _____ Date _____

Convince *me* that **you** understand the concept!

2nd Exam

I Given: $f(x)=\begin{cases} |x+3| & \text{if } x\le -2 \\ x^2+1 & \text{if } -2<x<1 \\ 16\sqrt{x+3}-30 & x\ge 1 \end{cases}$ (tot 50 pts)

a) It is obvious that $f(x)$ is continuous within each defined interval. Prove $f(x)$ is or is not continuous at ***both*** of the end points.

b) At each of the ***three*** points where the derivative of $f(x)$ is questionable, prove $f'(x)$ does or does not exist.

II Given two functions, $f(x)$ and $g(x)$. Explain how the IVT might be used to prove that the two functions have a common intersection point. **Do not use specific examples but be explicit.** (15 pts)

III Determine the following limits. (Hint: All limits *do* exist.) (5 pts ea.)

a) $\lim\limits_{x\to 0}\dfrac{\cos\left(x+\frac{\pi}{2}\right)}{x}$ b) $\lim\limits_{x\to\infty}\dfrac{3x^2-4}{2-7x-x^2}$ c) $\lim\limits_{x\to\infty}\dfrac{\sqrt{x}-4}{4-3\sqrt{x}}$ d) $\lim\limits_{x\to 0}\dfrac{\sin\left(x+\frac{\pi}{2}\right)-1}{x}$

e) $\lim\limits_{x\to 3^-}\dfrac{|x+3|}{\sqrt{x}}$ f) $\lim\limits_{x\to\infty} x\tan\frac{\pi}{x}$ g) $\lim\limits_{x\to-\infty}\left(\left(\frac{7}{5}\right)^x+\dfrac{3x^3+4}{7-4x^3}\right)$

Extra Credit ---------- 5 pts ----------

In a certain population, ⅔ of all the men are married but only ⅗ of the women are married. What fraction of the population is single? (Assume that all married people in the population have their spouses in that population also.)

I

A) $f(x) = x^2 + x$

$f^* = \lim_{h\to 0} \frac{(x+h)^2 + (x+h) - (x-h)^2 - (x-h)}{h}$

Total 20 pts

$\lim_{h\to 0} \frac{x^2+2xh+h^2+x+h-x^2+2xh-h^2-x+h}{h}$

$\lim_{h\to 0} \frac{4xh+2h}{h} = 4x+2$

B) $f(x) = \cos x$

$f^* = \lim_{h\to 0} \frac{\cos(x+h) - \cos(x-h)}{h}$

$= \lim_{h\to 0} \frac{\cos x\cos h - \sin x\sin h - \cos x\cos h - \sin x\sin h}{h}$

$= \lim_{h\to 0} \frac{-2\sin x \sin h}{h} = -2\sin x$

C) $f^* = 2f'$ since $f'(x) = 2x+1$ and $f'(x) = -\sin x$ in both cases f^* is twice of $f'(x)$.

II Intermediate value thm

Let f be continuous for $[a,b]$ and m be a value between $f(a)$ and $f(b)$ then $\exists\, x \in [a,b] \ni f(x) = m$.

when x goes from a to b, $f(x)$ changes from $f(a)$ to $f(b)$ it $f(x)$ takes on each value between $f(a)$ and $f(b)$. there must be no breaks, jumps or holes in $f(x)$ for that interval. (f is continuous)

15 pts.

III 5pts ea

A) $f(x) = x^4 - 4x - 6$ $f(0) = -6$ $f(\frac{1}{2}) = \frac{1}{16} - 2 - 6$

No root guaranteed. f takes on all values between -6 and $-\frac{15}{16}$ CC

B) $f(x) = x^2 - 2x - 2$ with a closed interval, ex. $[0,3]$ where $f(0) = -2$ and $f(3) = 1$ ∴ f has a zero

Note: there is a solution to $x^2 - 2x - 2 = 0$ but it is irrational so it cannot be solved with algebra

Numbers countable but uncountable

-8

IV 20 pts

Both radicals must be defined so both insides ≥ 0

$\frac{+0-0+}{-7 \quad -\frac{5}{2} \quad 2}$ at the same time $x \geq -7$ and $x \geq -\frac{5}{2}$ becomes $x \geq -\frac{5}{2}$ so Q must hold if $x \geq -\frac{5}{2}$

A) $f(x) = \begin{cases} \frac{\sqrt{2x+5} - \sqrt{x+7}}{x-2} & Q;\ x \neq 2 \\ K & x = 2 \end{cases}$

$\lim_{x\to 2} \frac{\sqrt{2x+5} - \sqrt{x+7}}{x-2} \cdot \frac{\sqrt{\ } + \sqrt{\ }}{\sqrt{\ } + \sqrt{\ }}$

$\lim_{x\to 2} \frac{2x+5-x-7}{(x-2)\sqrt{\ } + \sqrt{\ }}$

$\lim_{x\to 2} \frac{x-2}{x-2\sqrt{2x+5} + \sqrt{x+7}} = x \neq 2$

$\frac{1}{\sqrt{9} + \sqrt{9}} = \frac{1}{6}$

Set $K = \frac{1}{6}$. then $f(2) = \frac{1}{6}$

B) $\lim_{x\to 2} f(x) = \frac{1}{6} = f(2)$

$\lim_{x\to 2} f(x) = \frac{1}{6}$ and $\frac{1}{6} = \frac{1}{6}$

V. $f(x) = \sin x$

$$\lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h}$$

$$\lim_{h\to 0} \frac{\sin x\cos h + \sin h\cos x - \sin x}{h}$$

$$\lim_{h\to 0} -\sin x\left(\frac{1-\cos h}{h}\right) + \frac{\sin h}{h}\cos x = \cos x. \quad 10\text{pts}$$

($\frac{1-\cos h}{h} \to 0$, $\frac{\sin h}{h} \to 1$)

VI

4) $\lim_{x\to 0} \frac{\tan x - \sin x}{x}$

$$\lim_{x\to 0} \frac{\sin x}{x}\left(\frac{1}{\cos x} - 1\right) \quad 5\text{pts}$$

($\frac{\sin x}{x} \to 1$, $1 - 1 = 0$)

D) $\lim_{x\to 1} \sqrt{x^2-4}$

DNE since x may not approach 1 — 5pts

E) $\lim_{x\to -1} (2+x)^{\frac{1}{x+1}}$

$$\lim_{x\to -1} (1+(x+1))^{\frac{1}{x+1}}$$

let $h = x+1$

$$\lim_{h\to 0} (1+h)^{\frac{1}{h}} = e \quad 5\text{pts}$$

B) $\lim_{x\to -\infty} \frac{\sqrt{x^2+6}}{3x+2}$

$$\lim_{x\to -\infty} \frac{-x\sqrt{1+\frac{6}{x^2}}}{x(3+\frac{2}{x})}$$

$$\lim_{x\to -\infty} \frac{-\sqrt{1+\frac{6}{x^2}}}{3+\frac{2}{x}} = -\frac{1}{3} \quad 5.5\text{pts}$$

C) $\lim_{x\to 0} \frac{x+|x|}{\sqrt{x^2}}$

$$\lim_{x\to 0} \frac{x+|x|}{|x|}$$

$\lim_{x\to 0^-} \frac{x-x}{-x} = 0$ ←

$\lim_{x\to 0^+} \frac{x+x}{x} = 2$ ←

$\therefore \lim_{x\to 0} \frac{x+|x|}{\sqrt{x^2}}$ DNE

Extra.

$$|2x-1| = |4x+3|$$

I | 0 | II | 0 | III

$-\frac{3}{4}$ | 0 | $\frac{1}{2}$

5.5pts

if $x < -\frac{3}{4}$

$-2x+1 = -4x-3$

$2x = -4$

$x = -2$

if $x = -\frac{3}{4}$

$-2x+1 = 0$

$2x = 1$

$x = \frac{1}{2}$ Reject.

if $-\frac{3}{4} < x < \frac{1}{2}$

$-2x+1 = 4x+3$

$-2 = 6x$

$-\frac{1}{3} = x$

if $x = \frac{1}{2}$ $0 = 4x+3$ Reject.

if $x > \frac{1}{2}$

$2x-1 = 4x+3$

$-4 = 2x$

$-2 > x$ reject.

$\therefore$ solution

$x \in \{-2, -\frac{1}{3}\}$

96

100	3
90	6
80	6
70	4
60	3
55	1

105

55

$\frac{1835}{23} = 79.8$

I Need something easier to grade that II. It is too hard to decide what is a 5 person able.

Section I was too many pts for too easy a problem.

III 4 should be "counter example" because the interval was not closed as is required in the Def.

Make the change in IV as shown on extra.

Consider the change in the extras as shown

Advanced Placement Calculus

Name Calc 2nd Exam

Copy original problem.

Per _____ Date 10/11/96

Convince *me* that you understand the concept!

Chapters 1 – 4 Exam

20

I For a differentiable function, f, let $f^*(x)$ be the function given by: (25 pts tot)

$$f^*(x) = \lim_{h \to 0} \frac{f(x+h) - f(x-h)}{h}$$

A) Determine $f^*(x)$ when $f(x) = x^2 + x$ B) Determine $f^*(x)$ when $f(x) = \cos x$

C) Write an equation to express the relationship between the functions $f^*(x)$ and $f'(x)$

(Where $f'(x)$ denotes the usual derivative of f.)

15

II Explain the Intermediate Value Thm. No *not* use examples. (10 pts)

III What, concerning the zeros of $f(x)$, does the Intermediate Value Thm guarantee when: (5 pts ea)

A) $f(x) = \tan x - 3$ and the interval $\left[0, \frac{\pi}{2}\right)$ b) $f(x) = x^4 - 4x - 6$ and the interval $\left[0, \frac{1}{2}\right]$

IV Given $f(x) = \begin{cases} \dfrac{\sqrt{2x+5} - \sqrt{x+7}}{x-2} & \text{if } Q \\ K & \text{if } x = 2 \end{cases}$; $f(x)$ is a continuous function. (20 pts tot)

← Remove this

A) Let Q be the only added domain restriction of f. What is Q? B) Determine K.

V Using the definition of the derivative, find $f'(x)$ when $f(x) = \sin x$. (10 pts)

VI Limits. Determine the following limits, if they exist. Clearly explain reasoning. (5 pts ea)

A) $\lim_{x \to 0} \frac{\tan x - \sin x}{x}$ B) $\lim_{x \to -\infty} \frac{\sqrt{x^2+6}}{3x+2}$ C) $\lim_{x \to 0} \frac{x + |x|}{\sqrt{x^2}}$

D) $\lim_{x \to 1} \sqrt{x^2 - 4}$ E) $\lim_{x \to -1} (2+x)^{\frac{1}{x+1}}$

Extra Credit ---------- 5 pts ----------

Solve: $|2x - 1| = |4x + 3|$

$= |4x - 3|$ has 3 values.

I $f(x)=\sqrt{x^2+9}$

$f(g(x))=\sqrt{(g(x))^2-9} \overset{\text{set}}{=} x-\sqrt{x}$

Make no headway so I looked for an attempt 10

$[g(x)]^2-9=x^2-2x\sqrt{x}+x$

$[g(x)]^2=x^2-2x\sqrt{x}+x+9$

$g(x)=\pm\sqrt{x^2-2x\sqrt{x}+x+9}$

II $f(x)=\begin{cases}1+|x| & \text{if } x<0\\ x^3 & \text{if } 0\le x\le 2\\ 10-x & \text{if } x>2\end{cases}$

a) $f(0)=0$ ①

$\lim_{x\to 0^-} 1-x=1$

$\lim_{x\to 0^+} x^3=0 \neq$

Limit DNE

① ≠ ② ∴ discont at $x=0$

b) $f(2)=8$ ②

$\lim_{x\to 2^-} x^3=8$ ←

$=$ ② $=$

$\lim_{x\to 2^+} 10-x=8$ ←

$8=8$ ③

f is cont $x=2$

c) $f'(0)$ DNE since f is NOT cont at $x=0$

d) $\lim_{x\to 2}\frac{x^3-8}{x-2}$

$\lim_{x\to 2^-}\frac{(x-2)(x^2+2x+4)}{x-2}$

$4+4+4=12$

$\lim_{x\to 2^+}\frac{10-x-8}{x-2}$

$\lim_{x\to 2^-}\frac{-(x-2)}{x-2} \neq -1$

Limit DNE ① ≠ ② since limit is Def of Der DNE.

III $f(x)=\begin{cases}2mx+N & \text{if } x<2\\ 6 & \text{if } x=2\\ M-Nx & \text{if } x>2\end{cases}$

$\lim_{x\to 2^-} 2mx+N \overset{\text{sub}}{=} 4m+N \overset{\text{set}}{=} 6$

$\lim_{x\to 2^+} M-Nx = M-2N \overset{\text{set}}{=} 6$

$8m+2N=12$

$9m=18$

$m=2$ ∴

$8+N=6$

$N=-2$

$f(x)=\begin{cases}4x-2 & \text{if } x<2\\ 6 & \text{if } x=2\\ 2+2x & \text{if } x>2\end{cases}$

$f'(x)=\begin{cases}4 & \text{if } x<2\\ 2 & \text{if } x>2\end{cases}$

$\lim_{x\to 2^-}\frac{4x-2-6}{x-2}$ $\lim_{x\to 2^+}\frac{2+2x-6}{x-2}$

$\lim_{x\to 2^-}\frac{4(x-2)}{x-2}=4$ $\lim_{x\to 2^+}\frac{2(x-2)}{x-2}=2$

$4\neq 2$

∴ No 3pts at $x=2$.

IV $x^5-x^2+17=2x$

Let $f(x)=x^5-x^2-2x+17$

$\lim_{x\to-\infty} f(x)<0$

$\lim_{x\to+\infty} f(x)>0$

By IVT, There exists $\bar{x}$ such that $f(\bar{x})=0$, if $f(\bar{x})=0$ then the the equation has a solution.

V $f(x)=\frac{2x-2}{x^2+x-2}=\frac{2(x-1)}{(x-1)(x+2)}=\frac{2}{x+2}$

a) f is Discontinuous for $x\in\{1,-2\}$.

b) $\lim_{x\to 1} f(x)=\frac{2}{1+2}=\frac{2}{3}$

Reduceable Domain Restriction

$\lim_{x\to 2} f(x)$ DNE (L) <0 (R) >0

c) $\lim_{x\to\infty} f(x)=0$

∴ horizontal asymptote $y=0$

Vertical asymptote at $x=2$

Non reducable Domain Restriction

d) $a=2$

$b=+2$

VI

a) $\lim_{x\to 0} \frac{\sin x^2}{(\sin x)^2}$

$\lim_{x\to 0} \frac{\sin x^2}{x^2} \cdot \frac{x \cdot x}{\sin x} \cdot \frac{x}{\sin x}$

$1 \cdot 1 \cdot 1$

1

b) $\lim_{x\to 0} \frac{\sin 5(x-2)}{3(x-2)}$

$\lim_{x\to 0} \frac{\sin 5(x-2)}{\frac{3}{5} \cdot 5(x-2)} = \frac{5}{3}$

c) $\lim_{h\to 0} \frac{\cos(x+h) - \cos x}{h}$

This is def of derivative of $f(x) = \cos x$,

$f'(x) = -\sin x$ so that is the limit.

d) $\lim_{x\to\infty} \frac{\sqrt{2x^2+1}}{x+2}$

$\lim_{x\to -\infty} \frac{-x\sqrt{2+\frac{1}{x^2}}}{x(1+\frac{2}{x})} = -\sqrt{2}$

e) $\lim_{x\to 4} \frac{x^3-64}{\sqrt{x}-2} = 192$

$\lim_{x\to 4} \frac{(x-4)(x^2+4x+16)}{\sqrt{x}-2}$

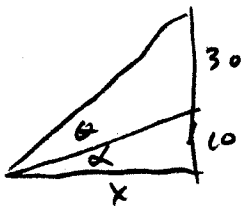
$\lim_{x\to 4} \frac{(\sqrt{x}-2)(\sqrt{x}+2)(x^2+4x+16)}{(\sqrt{x}-2)}$

$4(16+16+16)$

48

192

Extra



$\tan(\theta+\alpha) = \frac{40}{x}$ $\tan\alpha = \frac{10}{x}$

$\frac{\tan\theta + \tan\alpha}{1 - \tan\theta\tan\alpha} = \frac{40}{x}$

$\frac{\frac{10}{x} + \tan\theta}{1 - \frac{10}{x}\tan\theta} = \frac{40}{x}$

$\frac{10 + x\tan\theta}{x - 10\tan\theta} = \frac{40}{x}$

$10x + x^2\tan\theta = 40x - 400\tan\theta$

$\tan\theta[x^2 + 400] = 30x$

$\tan\theta = \frac{30x}{x^2+400}$

10/14/87

Looks like they expected a "previous" exam maybe they can focus on concepts

100	0
90	0
80	4
70	4
60	3
59	6

87

$\frac{1122}{17} = 66\%$ 39

one typo VI b

Advanced Placement Calculus

Copy original problem.

Convince *me* that you understand the concept!

Name Calc 1-4

Per _____ Date 1997

Chapters 1 – 4 Exam

I Let $f(x) = \sqrt{x^2 + 9}$. Determine two different functions $g(x)$ so that $f(g(x)) = x - \sqrt{x}$. (15 pts)

II Let $f(x) = \begin{cases} 1 + |x| & \text{if } x < 0 \\ x^3 & \text{if } 0 \le x \le 2 \\ 10 - x & \text{if } x > 2 \end{cases}$ (15 pts)

a) Prove $f(x)$ is or is not continuous at $x = 0$.

b) Prove $f(x)$ is or is not continuous at $x = 2$.

c) Prove $f'(0)$ exists or does not exist.

d) Prove $f'(2)$ exists or does not exist.

III Let $f(x) = \begin{cases} 2mx + n & \text{if } x < 2 \\ 6 & \text{if } x = 2 \\ m - nx & \text{if } x > 2 \end{cases}$ (15 pts)

a) Determine the value of the constants m and n so that $f(x)$ will be continuous.

b) Rewrite $f(x)$ with the proper values of m and n and then determine $f'(x)$

IV Verify with a careful and complete explanation that the equation $x^5 - x^2 + 17 = 2x$ has at least one solution. (10 pts)

Prove That The Derivative DNE

V Let $f(x) = \dfrac{2x - 2}{x^2 + x - 2}$ (20 pts tot)

MLK EXAM

a) For what values of x is $f(x)$ discontinuous?

b) At each point of discontinuity found in part (a), determine whether $f(x)$ has a limit and, if so, give the value of the limit.

c) Write the equation for each vertical and horizontal asymptote to the graph of $f(x)$. Justify, of course.

d) A rational function $g(x) = \dfrac{a}{x + b}$ is such that $g(x) = f(x)$ wherever $f(x)$ is defined. Find the values of a and b.

VI Limits. Be very sure your method is justified. (Hint: All limits exist) (5 pts ea)

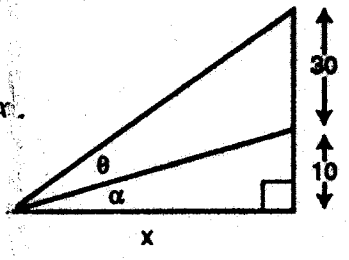
a) $\lim\limits_{x \to 0} \dfrac{\sin x^2}{(\sin x)^2}$ b) $\lim\limits_{x \to 0} \dfrac{\sin(5x - 10)}{3x - 6}$ c) $\lim\limits_{h \to 0} \dfrac{\cos(x + h) - \cos x}{h}$

2

d) $\lim\limits_{x \to -\infty} \dfrac{\sqrt{2x^2 + 1}}{x + 2}$ e) $\lim\limits_{x \to 4} \dfrac{x^3 - 64}{\sqrt{x} - 2}$

Extra Credit ---------- 5 pts ----------

Determine $\tan\theta$ in terms of x.



I $f(x) = \frac{3x^2-2}{x+2}$ $f'(x) = \frac{(x+2)6x-(3x^2-2)}{(x+2)^2}$

$f(1) = \frac{3-2}{3} = \frac{1}{3}$ $= \frac{6x^2+12x-3x^2+2}{x+2}$

$= \frac{3x^2+12x+2}{(x+2)^2}\Big|_{x=1}$

$\frac{3+12+2}{9}$

$\frac{17}{9} = m$

$y-\frac{1}{3} = \frac{17}{9}(x-1)$

$y-\frac{1}{3} = \frac{17}{9}x - \frac{17}{9}$

$y = \frac{17}{9}x - \frac{14}{9}$

II $f(x) = x^2-2x$

$\lim_{x\to 3} \frac{x^2-2x-(9-c)}{x-3}$

$\lim_{x\to 3} \frac{x^2-2x-3}{x-3}$

$\lim_{x\to 3} \frac{(x-3)(x+1)}{x-3} = 4$

III f cont at $x=2$, $x=3$

$f(2)=0$ $f(3)=-1$

$g(x) = \frac{2(f(x))^2+x-2}{4f(x)+x^2-5}$

A) $g(3) = \frac{2\cdot 1+3-2}{4(-1)+9-5} = \frac{}{0}$

g is not defined at $x=3$

∴ g not cont at $x=3$.

B) $g(2) = \frac{2\cdot 0+2-2}{4\cdot 0+4-5} = 0$

$\lim_{x\to 2} \frac{2(f(x))^2+x-2}{4f(x)+x^2-5}$

Since f is cont at $x=2$,

$\lim_{x\to 2} f(x) = f(2) = 0$

$\frac{2\cdot 0+2-2}{4\cdot 0+4-5} = 0$

$g(2) = 0$

$\lim_{x\to 2} g(x) = 0$

$0=0$ ∴ g is cont at $x=2$

IV $f(x) = 2\sin 3x \cos x$

A) $f(-\pi) = 2\cdot 0 + \cos(-\pi) = -1$

$f(\frac{\pi}{3}) = 2\sin\pi + \cos\frac{\pi}{3}$

$0 + \frac{1}{2} = \frac{1}{2}$

There is no $\bar{x}$ such that $f(\bar{x}) = -\frac{3}{4}$ because the Smallest $2\sin 3x \cos x$ can be is -2 and the smallest x can be is -1 and -3 is not less than $-3\frac{1}{4}$.

B) use values above (Part A)

Since $-1 \le -\frac{3}{4} < \frac{1}{2}$

and since f is cont, the IVT guarantees the existence of $\bar{x}$

V $f(x) = \begin{cases} px+Q & \text{if } x < \frac{1}{2} \\ 3x^2 & \text{if } x \geq \frac{1}{2} \end{cases}$ f is continuous

1) $\lim_{x \to \frac{1}{2}^-} f(x)$

$\lim_{x \to \frac{1}{2}^-} px+Q = \frac{1}{2}p+Q$

2) $\lim_{x \to \frac{1}{2}^+} f(x)$

$\lim_{x \to \frac{1}{2}^+} 3x^2 = \frac{3}{4}$

c) $\lim_{x \to \frac{1}{2}} f(x) = f(\frac{1}{2})$

$\frac{1}{2}p+Q = \frac{3}{4}$

$Q = -\frac{1}{2}p + \frac{3}{4}$

$Q = -\frac{1}{2}p + \frac{3}{4}$

$Q = -\frac{1}{2}(3) + \frac{3}{4}$

$= -\frac{3}{2} + \frac{3}{4}$

$Q = -\frac{3}{4}$

8) TS

$\lim_{x \to \frac{1}{2}^-} \frac{px+Q - \frac{3}{4}}{x - \frac{1}{2}}$

$\lim_{x \to \frac{1}{2}^-} \frac{px - \frac{1}{2}p + \frac{3}{4} - \frac{3}{4}}{x - \frac{1}{2}}$

$\lim_{x \to \frac{1}{2}^-} \frac{p(x-\frac{1}{2})}{x-\frac{1}{2}} = p$

$\lim_{x \to \frac{1}{2}^+} \frac{3x^2 - \frac{3}{4}}{x - \frac{1}{2}}$

$\lim_{x \to \frac{1}{2}^+} \frac{12x^2 - 3}{4(x-\frac{1}{2})}$

$\lim_{x \to \frac{1}{2}^+} \frac{3(2x-1)(2x+1)}{2(2x-1)} = \frac{3}{2}(1+1) = 3$

in order for $f'(\frac{1}{2})$ to exist
Both above limits must be =.

$\therefore p = 3$

10/27/98

McNally 95
Shannon 93
Plant 91
Johnson 91
Ruiz 89
Payden 89
Work 85
Deas 83
Snyder 77
Schaulis 40

10/21/93:

Ken Whiseman 86
Meghan Forehand 70

10/24/94

Dustin Baker 84
Erin Serris 30

10/18/95

Steve Soggs 100
Sundeep Wang 99
Brooke Barton 87
Tracy Sacca 65

10/28/97

Mike Hong 95
Amber Sirois 95
Brian Horn 94
Rebecca Wells 91
JD Pitnes 91
Nick Ouda 85
Kristor Vlad 75
Eli Deberec 75
Jeff Baker 75
Ken Tsao 60
Lacey Palmer 68

10/20/99 ave 83.5

Dana Nicholas 91
Dylan Ballard, Josh Klobas, Andrea Marlowe } 80
Shannon Chandler 86
Sarah Sactions, Tim Garvey } 85
Jason Rachel 81
Tara Holstein 79
Rachel Dutra 78
Danni Chen 75
Rajvir Basra 72

Advanced Placement Calculus Name 90/93/94/95/99

Copy original problem. Per _____ Date _____

Convince *me* that **you** understand the concept!

No Calculators. Each Section 20 pts, total.

Chapters 1 – 4 Make Up Exam

I Determine the equation of the line tangent to $f(x)$ at $x=1$ given $f(x)=\dfrac{3x^2-2}{x+2}$.

(Note: You may use the quotient rule for derivatives: $\left(\dfrac{u}{v}\right)' = \dfrac{v\ du - u\ dv}{v^2}$.)

II Given $f(x)=x^2-2x$. Using the definition of the derivative, prove: $f'(3)=4$

III Given: $f(x)$ is a function which is continuous at $x=3$ and at $x=2$

$f(2)=0\ ;\ f(3)=-1$

$g(x)=\dfrac{2(f(x))^2+x-2}{4f(x)+x^2-5}$

a) Prove: $g(x)$ is or is not continuous at $x=3$.

b) Prove: $g(x)$ is or is not continuous at $x=2$.

IV Given: $f(x)=2\sin(3x)+\cos x$. Justify your conclusions.. Determine if a X exists such that ...

a) $-\pi \le X \le \dfrac{\pi}{3}$ for which $f(X)=-\dfrac{13}{4}$

b) $-\pi \le X \le \dfrac{\pi}{3}$ for which $f(X)=-\dfrac{3}{4}$

V Given the continuous function: $f(x)=\begin{cases} Px+Q & \text{if } x<\dfrac{1}{2} \\ 3x^2 & \text{if } x\ge\dfrac{1}{2} \end{cases}$

a) Find $\lim\limits_{x\to\frac{1}{2}^-} f(x)$

b) Find $\lim\limits_{x\to\frac{1}{2}^+} f(x)$

c) Since $f(x)$ is continuous, what can be concluded from steps "a" and "b" above? (ie. solve for Q.)

d) Determine P and Q so that $f'\left(\frac{1}{2}\right)$ exists.

I $f(x) = \begin{cases} 1-x & \text{if } x<0 \\ 1+|x| & \text{if } x<0 \\ x^3 & \text{if } 0 \le x \le 2 \\ 10-x & \text{if } x>2 \end{cases}$

a) $f(0) = 0$ ①

$\lim_{x\to 0^-} 1-x = 1$ ②

$\lim_{x\to 0^+} x^3 = 0$ ≠ ②

Not cont.

Fails at Step 2.

b) $f(2) = 8$ ①

$\lim_{x\to 2^-} x^3 = 8$ ②

$\lim_{x\to 2^+} 10-x = 8$ ≠ ②

f is cont at $x=2$ ← ③

c) f not cont. at $x=0$

∴ DER DNE

d) $\lim_{x\to 2^-} \frac{x^3-8}{x-2}$

$\lim_{x\to 2} \frac{(x-2)(x^2+2x+4)}{x-2}$

$4+4+4$

12 ←

$\lim_{x\to 2^+} \frac{10-x-8}{x-2}$ ≠ ∴ DNE

$\frac{-(x-2)}{x-2} = -1$ at $x=2$

II $f(x) = x^3-x^2-4x+4$

$x^2(x-1)-4(x-1)$

$(x^2-4)(x-1)$

$(x-2)(x+2)(x-1)$

a) zeros of f $x \in \{1, \pm 2\}$

b) $f(a) = b = a^3-a^2-4a+4$

$f'(x) = 3x^2-2x-4$

$f'(a) = 3a^2-2a-4$

$y-b = (3a^2-2a-4)(x-a)$

This line passes through $(0,-8)$ so $x=0, y=-8$

$-8-b = -3a^3+2a^2+4a$

$b = 3a^3-2a^2+4a-8 \overset{\text{set}}{=} a^3-a^2-4a+4$

$2a^3-a^2-12=0$

	2	-1	0	-12
2	2	3	6	0

$(x-2)(2a^2+3a+6)$ $9-48<0$ ∴ no other roots.

$a=2 \quad b = 8-4-8+4 = 0$

	1	-1	-4	4
-2	1	-2	-2	6
-2	1	-3	1	$=m$

$y-6 = 1(x+1)$

$y = x+7$

III $x^5-x^2+17 = 20$

Let $f(x) = x^5-x^2-2x+17$

$\lim_{x\to-\infty} f(x) \to -\infty$

$\lim_{x\to+\infty} f(x) \to +\infty$

∴ $\exists X \to f(X) = 0$ by IVT

Since f has a zero, this equation has a solution.

IV $f(x) = \sqrt{x^2+9}$ $f(g(x)) = x-\sqrt{x}$

$f(g(x)) = \sqrt{(g(x))^2+9} \overset{\text{set}}{=} x-\sqrt{x}$

$(g(x))^2+9 = x^2-2x\sqrt{x}+x$

$(g(x))^2 = x^2-2x\sqrt{x}+x-9$

$g(x) = \pm\sqrt{x^2-2x\sqrt{x}+x-9}$

U

a) $\lim_{x\to 0} \frac{\sin x^2}{(\sin x)^2}$

$\lim_{x\to 0} \frac{\sin x^2}{x^2} \cdot \frac{x}{\sin x} \cdot \frac{x}{\sin x}$

$1 \cdot 1 \cdot 1$

1

b) $\lim_{x\to 2} \frac{\sin(5x-10)}{3x-6}$

$\lim_{x\to 2} \frac{\sin 5(x-2)}{3(x-2)} \cdot \frac{5}{5}$

$\lim_{x\to 2} \frac{\sin 5(x-2)}{5(x-2)} \cdot \frac{5}{3}$

$\frac{5}{3}$

c) $\lim_{h\to 0} \frac{\cos(x+h) - \cos x}{h}$

Def of limit cosine...

$-\sin x$

D) $\lim_{x\to -\infty} \frac{\sqrt{2x^2+1}}{x+2}$

$\lim_{x\to -\infty} \frac{-x\sqrt{2+\frac{1}{x^2}}}{x(1+\frac{2}{x})}$

$\lim_{x\to -\infty} \frac{-\sqrt{2+\frac{1}{x^2}}}{1+\frac{2}{x}} = -\sqrt{2}$

E) $\lim_{x\to 4} \frac{x^3-64}{\sqrt{x}-2}$

$\lim_{x\to 4} \frac{(x-4)(x^2+4x+16)}{\sqrt{x}-2}$

$\lim_{x\to 4} \frac{(\sqrt{x}-2)(\sqrt{x}+2)(x^2+4x+16)}{(\sqrt{x}-2)}$

$(2+2)(16+16+16)$

$4(48)$

192

F) $\lim_{x\to -\infty} \frac{2x-20}{\sqrt{x^2+100}}$

$\lim_{x\to -\infty} \frac{x(2-\frac{20}{x})}{-x\sqrt{1-\frac{100}{x^2}}}$

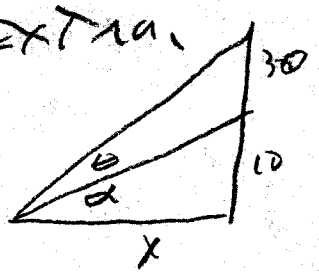
$\lim_{x\to -\infty} \frac{2-\frac{20}{x}}{-\sqrt{1-\frac{100}{x^2}}} = -2$

G) $\lim_{x\to \frac{\pi}{2}} \frac{1-\sin x}{\cos^2 x} \cdot \frac{1+\sin x}{1+\sin x}$

$\lim_{x\to \frac{\pi}{2}} \frac{1-\sin^2 x}{\cos^2 x} \cdot \frac{1}{1+\sin x}$

$\lim_{x\to \frac{\pi}{2}} 1 \cdot \frac{1}{1+\sin x} = 1 \cdot \frac{1}{1+1} = \frac{1}{2}$

EXTRA.



$\tan(\theta+\alpha) = \frac{30}{x}$

$\tan\alpha = \frac{10}{x}$

$\tan(\theta+\alpha) = \frac{\tan\theta + \tan\alpha}{1 - \tan\theta\tan\alpha}$

$\frac{40}{x} \overset{SET}{=} \frac{\tan\theta + \frac{10}{x}}{1 - \frac{10}{x}\tan\theta}$

$= \frac{\frac{x\tan\theta + 10}{x}}{\frac{x - 10\tan\theta}{x}}$

$= \frac{x\tan\theta + 10}{x - 10\tan\theta}$

$= \frac{x\tan\theta + 10}{x - 10\tan\theta}$

$40x - 400\tan\theta = x^2\tan\theta + 10x$

$(x^2+400)\tan\theta = 30x$

$\tan\theta = \frac{30x}{x^2+400}$

100	6
90	1
80	6
70	6
60	5
59	6

68.5

Advanced Placement Calculus

Name Calc Exam #2

Per _____ Date 1999

Copy original problem.

Convince *me* that **you** understand the concept!

No Calculators.

Chapters 1 – 4 Exam

I Let $f(x) = \begin{cases} 1 + |x| & \text{if } x < 0 \\ x^3 & \text{if } 0 \le x \le 2 \\ 10 - x & \text{if } x > 2 \end{cases}$ (15 pts tot)

a) Prove $f(x)$ is or is not continuous at $x = 0$.

b) Prove $f(x)$ is or is not continuous at $x = 2$

c) Prove $f'(0)$ exists or does not exist.

d) Prove $f'(2)$ exists or does not exist.

II Given the function f defined by $f(x) = x^3 - x^2 - 4x + 4$ (25 pts tot)

a) Find the zeros of f.

b) Write the equation of the line tangent to the graph of f at $x = -1$.

c) The point (a,b) is on the graph of f and the line tangent to the graph at (a,b) passes through the point $(0, -8)$ which is not on the graph of f. Find the values of a and b.

III Verify with a careful and *complete explanation* that the equation $x^5 - x^2 + 17 = 2x$ has at least one solution. (10 pts)

IV Let $f(x) = \sqrt{x^2 + 9}$. Determine $g(x)$ so that $f(g(x)) = x - \sqrt{x}$. (15 pts)

V Limits. Be very sure your method is justified. (Hint: All limits exist) (5 pts ea)

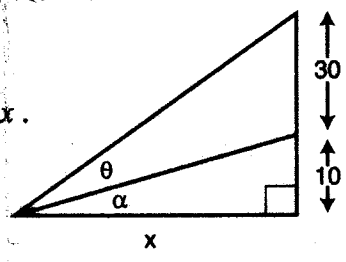
change to 2π/3

a) $\lim_{x \to 0} \dfrac{\sin x^2}{(\sin x)^2}$ b) $\lim_{x \to 2} \dfrac{\sin(5x-10)}{3x-6}$ c) $\lim_{h \to 0} \dfrac{\cos(x + h) - \cos x}{h}$

d) $\lim_{x \to -\infty} \dfrac{\sqrt{2x^2 + 1}}{x + 2}$ e) $\lim_{x \to 4} \dfrac{x^3 - 64}{\sqrt{x} - 2}$ f) $\lim_{x \to -\infty} \dfrac{2x-20}{\sqrt{x^2-100}}$ g) $\lim_{x \to \frac{\pi}{2}} \dfrac{1-\sin x}{\cos^2 x}$

Extra Credit ---------- 5 pts ----------

Determine $\tan\theta$ in terms of x.



"Continuous Enjoyment -- 2nd Edition"

For each problem, select the proper multiple choice response. Include with your work a detailed explanation of your method and justification.

1) If $\sec x$ is positive then it can be expressed in terms of $\sin x$ as

a) $\frac{1}{\sin x}$ b) $\frac{1}{1-\sin^2 x}$ c) $\frac{1}{\sqrt{1-\sin^2 x}}$ d) $\frac{\sin x}{1-\sin x}$ e) $\frac{1}{1-\sin x}$

2) Which of the following functions is even?

I. $f(x)=\sin(x^2-1)$ II. $g(x)=\cos(x^3-2)$ III. $h(x)=(x-3)^2$

a) I only b) II only c) III only d) II and III only e) I, II, and III

3) Which of the following functions has a vertical asymptote at $x=\frac{\pi}{2}$?

I. $f(x)=\sin\left(x-\frac{\pi}{2}\right)$ II. $g(x)=\tan x$ III. $h(x)=\cot x$

a) I only b) II only c) III only d) II and III only e) I and III only

4) If the angle from the x-axis to a given line is 30°, then the slope of the line is:

a) $\sqrt{3}$ b) $\frac{\sqrt{3}}{3}$ c) $\frac{1}{2}$ d) $\frac{\sqrt{3}}{2}$ e) 1

5) The period of the function $y=3\tan\left(\frac{2}{3}x-\pi\right)$ is

a) $\frac{2\pi}{3}$ b) π c) $\frac{\pi}{2}$ d) $\frac{3\pi}{2}$ e) 3π

6) If $f(x)=\frac{\sin x-x}{1-\cos x}$ then $f(-x)=$

a) $\frac{\sin x+x}{1+\cos x}$ b) $\frac{\sin x-x}{1-\cos x}$ c) $\frac{x-\sin x}{1-\cos x}$ d) $\frac{-\sin x-x}{1-\cos x}$ e) $\frac{\sin x-x}{1+\cos x}$

7) Which of the following functions is neither even nor odd?

a) $f(x)=\tan x$ b) $f(x)=\sin^2 x$ c) $f(x)=\cos|x|$ d) $f(x)=\frac{1-\cos x}{\sin x}$ e) $f(x)=\sec x+\csc x$

8) If $f(x)=x^2$ then $\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}=$

a) a^2 b) $2a$ c) a^2+2a d) a^2-2a e) 0

9) The equation of the tangent line to $f(x)=-\frac{3}{x}$ at the point where $x=3$ is:

a) $y=\frac{1}{3}x-2$ b) $y=-\frac{1}{3}x$ c) $y=\frac{1}{3}$ d) $y=-3x+8$ e) $y=-3x+10$

10) $\lim_{x\to 1}\frac{x-1}{x^2+2x-3}=$

a) 0 b) no limit c) ∞ d) $\frac{1}{2}$ e) $\frac{1}{4}$

11) If $f(x) = 2 - \left|4 + \frac{x}{2}\right|$ then $f(x)$ is not differentiable at $x =$

a) -2 b) 2 c) 0 d) -8 e) 8

12) $\lim\limits_{x \to -\frac{1}{3}} \dfrac{9x^2 - 1}{3x + 1} =$

a) 0 b) Nonexistent c) -1 d) -2 e) $-\frac{1}{3}$

13) $\lim\limits_{x \to 2} \dfrac{\sqrt{x+14} - 4}{x - 2}$

a) 0 b) 8 c) $\frac{1}{8}$ d) 4 e) $\frac{1}{4}$

14) Let $f(x) = \dfrac{x-7}{\sqrt{x-3} - 2}$ which of the following statements are true?

I. $\lim\limits_{x \to 7} f(x) = 4$ II. $f(7)$ is undefined. III. $\lim\limits_{x \to 7} f(x) = f(7)$

a) I only b) II only c) III only d) I and III only e) I and II only

15) Given $f(x) = 3x - 5$ and $g(x) = 3x^3 - 5x^2 - 3x + 5$. Find $\lim\limits_{x \to \frac{5}{3}} \dfrac{g(x)}{f(x)}$.

a) $\frac{34}{9}$ b) $\frac{16}{9}$ c) 0 d) $\frac{5}{3}$ e) $\frac{16}{3}$

16) If $f(x) = \dfrac{3x-1}{2x}$ then $\lim\limits_{x \to 0} f(x) =$

a) 0 b) ∞ c) -1 d) $-\frac{1}{2}$ e) DNE

17) Evaluate: $\lim\limits_{x \to 5} \sqrt{\dfrac{x^2 - 9}{x - 3}}$

a) 8 b) $2\sqrt{2}$ c) 5 d) $\sqrt{2}$ e) 2

18) Given $f(x) = \begin{cases} 3x - 2 & \text{if } x \geq 5 \\ -x^2 + 3 & \text{if } 0 < x < 5 \\ -x^3 + 1 & \text{if } x \leq 0 \end{cases}$. Find $\lim\limits_{x \to 5^-} f(x)$.

a) 13 b) 28 c) -22 d) -124 e) None of These

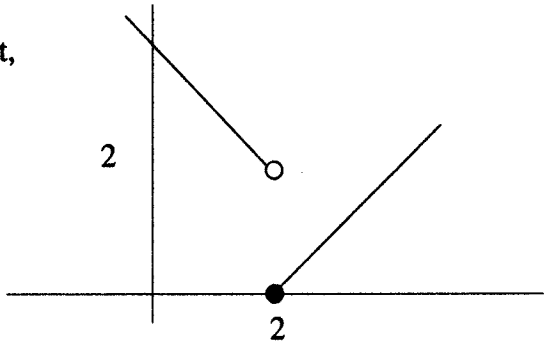
19) Given $f(x) = \dfrac{|2x-4|}{x-2}$. Find $\lim\limits_{x \to 2^+} f(x)$.

a) DNE b) 0 c) 2 d) 1 e) 3

20) Given $f(x)=\begin{cases}\sqrt{(x^2-4)} & : \ x\le -2 \text{ or } x\ge 2 \\ x^2-2 & ; \ -2<x<2\end{cases}$. Find $\lim_{x\to -2^-} f(x)$.

a) 2 b) -2 c) 4 d) 0 e) 1

21) From the graph of $f(x)$ shown on the right, which is necessarily true?



I. $\lim_{x\to 2^-} f(x)$ DNE

II. $\lim_{x\to 2^+} f(x)=0$

III. $\lim_{x\to 2} f(x)=0$

a) I only b) II only c) III only d) I and III only e) I and II only

22) Given $f(x)=\dfrac{x^2+2x}{x(x+2)^2(x-1)^4}$, then $\lim_{x\to a} f(x)=\infty$ for $a=$

a) $0,-2,1$ b) $0,1$ c) $1,-2$ d) $-1,2,0$ e) $-1,2$

23) The equation(s) of the vertical asymptote(s) of $f(x)=\dfrac{2x-5}{25-4x^2}$ is (are)

a) $y=\frac{5}{2}$ b) $x=-\frac{5}{2}$ c) $x=\frac{5}{2},\ x=-\frac{5}{2}$ d) $y=\frac{5}{2},\ y=-\frac{5}{2}$ e) $y=-\frac{5}{2}$

24) $\lim_{x\to 5^-} \dfrac{x^2-3x-10}{x^2-25}$

a) $\frac{7}{10}$ b) DNE c) $\frac{2}{5}$ d) 1 e) 0

25) Which of the following functions has a vertical asymptote at $x=2$?

I. $f(x)=\dfrac{1}{x-2}$ II. $g(x)=\dfrac{x-2}{x-2}$ III. $h(x)=\begin{cases}\dfrac{1}{x-2} & ; \ x<2 \\ x+5 & ; \ x\ge 2\end{cases}$

a) I only b) II only c) III only d) I and III only e) I and II only

26) The equation of the horizontal asymptote of $f(x)=\dfrac{3x^2-2x+5}{7x-2x^2-1}$ is:

a) $y=\frac{3}{7}$ b) $x=\frac{3}{7}$ c) $y=-\frac{3}{2}$ d) $y=-5$ e) $x=-\frac{3}{2}$

27) Find $\lim_{x\to -\infty} \dfrac{x^2+4}{x^4-1}$.

a) 0 b) $\frac{1}{2}$ c) $-\frac{1}{2}$ d) ∞ e) DNE

28) The equation of the horizontal asymptote to $xy - 2y = 3x - 4$ is:

a) $y = 2$ b) $x = 2$ c) $y = 3$ d) $x = 3$ e) none of these

29) $\lim_{x \to -\infty} \left(-5x + \frac{3}{2x^3} \right) =$

a) 0 b) ∞ c) $-\infty$ d) $\frac{3}{2}$ e) -5

30) The graph of the function $f(x) = \frac{x^2 + 2x}{x^2 - 2x}$ has asymptotes at:

a) $x = 0, x = 2, y = 1$ b) $x = 0, x = 2, y = 0$ c) $x = -2, y = 1$ d) $x = 0, x = -2, y = 1$ e) $x = 2, y = 1$

31) For what value(s) of x does $f(x) = \frac{x^2 - 2}{1 - x - x^2}$ cross its horizontal asymptote?

a) 0 b) $\pm\sqrt{2}$ c) -1 d) 1 e) -2

32) If $f(x) = \frac{x + 4}{2x - 1}$ which of the following values of x guarantees that $\left| f(x) - \frac{1}{2} \right| < \frac{1}{10}$?

a) $x < 20$ b) $x > \frac{46}{45}$ c) $x < 23$ d) $x > 23$ e) $x > 20$

33) Find $\lim_{x \to \infty} \frac{\sqrt{x^2 + 4x + 4}}{2x + 3}$

a) ∞ b) 0 c) $\frac{1}{2}$ d) $\frac{1}{3}$ e) 2

34) Given $f(x) = \begin{cases} 2 - 3x & \text{if } x < 1 \\ -1 & \text{if } x = 1 \\ 2x^2 - 5 & \text{if } x > 1 \end{cases}$. Which of the following statements are true?

I. $\lim_{x \to 1^-} f(x) = -1$ II. $f(1) = -1$ III. $\lim_{x \to 1} f(x) = f(1)$

a) I only b) II only c) III only d) I and II only e) I, II, III

35) Determine the value of $a + b$ so that the function $f(x) = \begin{cases} -3ax^2 - 1 & ; x > 1 \\ 3b - 2x & ; x \le 1 \end{cases}$ is continuous at $x = 1$.

a) $\frac{1}{3}$ b) 1 c) $-\frac{1}{3}$ d) -1 e) 0

36) Determine the value of k such that $f(x) = \begin{cases} 3kx - 5 & ; x > 2 \\ 4x - 5k & ; x \le 2 \end{cases}$ is continuous.

a) 3 b) $\frac{13}{11}$ c) $\frac{3}{11}$ d) $-\frac{3}{11}$ e) -3

37) Choose b so $f(x) = \begin{cases} \dfrac{x^2+x-2}{x-1} & ; \ x<1 \text{ or } x>1 \\ b & x=1 \end{cases}$ is continuous.

a) $b \ \varepsilon \ \Re$ b) $b \ \varepsilon \ \{\ \}$ c) 3 d) 1 e) -1

38) Which of the following statements is **not** true?

a) If $f(x)$ is continuous at $x = a$, then $\lim\limits_{x \to a} f(x)$ exists.

b) If $f(x)$ is continuous at $x = a$, then it is defined at $x = a$.

c) If $f(x)$ is defined at $x = a$, then $f(x)$ is continuous at $x = a$.

d) If $\lim\limits_{x \to a} f(x) = f(a)$, then $f(x)$ is continuous at $x = a$

e) If $f(x)$ is a polynomial, then it is continuous at $x = a$.

39) Which of the following functions are continuous on $(-\infty, \infty)$?

I. $f(x) = x^2 + 4x - 3$ II. $f(x) = \dfrac{x+1}{x-1}$ III. $f(x) = \begin{cases} x^2+2x & ; \ x<2 \\ 2x+4 & ; \ x \geq 2 \end{cases}$

a) I only b) II only c) III only d) I and III only e) I and II only

40) The function $f(x) = \begin{cases} -3x^2 - 1 & ; \ x > -2 \\ -13 & ; \ x = -2 \\ -x^2 - 17 & ; \ x < -2 \end{cases}$ is discontinuous at $x = -2$ because:

I. $\lim\limits_{x \to -2^-} f(x) \neq \lim\limits_{x \to -2^+} f(x)$ II. $\lim\limits_{x \to -2^-} f(x)$ DNE III. $f(-2) \neq \lim\limits_{x \to -2^-} f(x)$

a) I only b) II only c) III only d) I and III only e) I, II, III

41) $\lim\limits_{x \to 0} \dfrac{2\sin 3x}{5x} =$

a) $\dfrac{2}{5}$ b) $\dfrac{6}{5}$ c) 0 d) DNE e) $\dfrac{3}{5}$

42) $\lim\limits_{x \to \frac{\pi}{4}} \dfrac{2x - \frac{\pi}{2}}{\cos 2x} =$

a) 0 b) DNE c) $\dfrac{\pi}{4}$ d) -1 e) 1

43) $\lim\limits_{x \to \infty} \dfrac{\sin x}{2x} =$

a) 0 b) 1 c) ∞ d) $\dfrac{1}{2}$ e) 2

44) $\lim\limits_{x \to 0} \dfrac{\tan 2x}{x} =$

a) $\dfrac{1}{2}$ b) 0 c) ∞ d) 1 e) 2

45) $\lim_{x \to \frac{\pi}{2}} \frac{\sin\left(\frac{\pi}{2} - x\right)}{\frac{\pi}{2} - x} =$

a) DNE b) -1 c) 0 d) ∞ e) 1

46) $\lim_{x \to 0} \left(\sin \frac{1}{x}\right)\left(\cos \frac{1}{x}\right) =$

a) 0 b) 1 c) $\frac{1}{2}$ d) -1 e) DNE

47) Which of the following functions is continuous and differentiable everywhere?

I. $f(x) = 2x^{\frac{3}{2}}$ II. $g(x) = \frac{3}{2x}$ III. $h(x) = 5x^{\frac{2}{5}} - 3$

a) I only b) II only c) III only d) I and II only e) I, II, III

48) Which of the following functions is differentiable everywhere?

I. $f(x) = 2x^{\frac{3}{2}}$ II. $g(x) = \frac{3}{2x}$ III. $h(x) = 5x^{\frac{2}{5}} - 3$

a) I only b) II only c) III only d) I and II only e) I, II, III

49) Find the value of $a + b$ such that f is differentiable at $x = -1$ if $f(x) = \begin{cases} 3x^2 & ; x < -1 \\ ax + b & ; x \geq -1 \end{cases}$.

a) 3 b) -9 c) -6 d) -3 e) 6

50) If $f(x) = \begin{cases} 3x^2 + 4 & ; x < 2 \\ bx^3 & ; x \geq 2 \end{cases}$, find b so that $f(x)$ is continuous.

a) 3 b) $\frac{1}{2}$ c) -1 d) 1 e) 2

51) If $f(x) = \begin{cases} 3x^2 + 4x & ; x \leq 1 \\ 2x^3 + bx + c & ; x > 1 \end{cases}$, find b and c so that $f(x)$ has a derivative at $x = 1$.

a) b=4, c=0 b) b=1, c=4 c) b=4, c=1 d) b=1, c=0 e) b=1, c=1

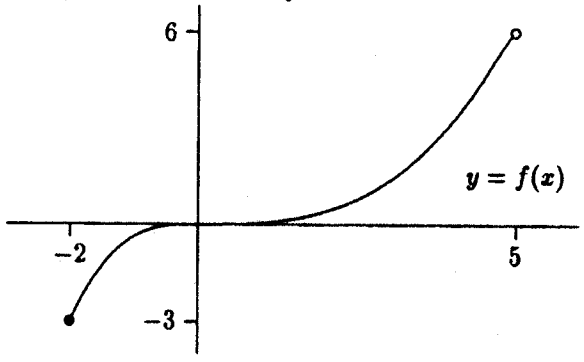
52) $f(x) = |x^3|$ then which of the following are true?

I. $f(x)$ is symmetric with respect to the origin.
II. $f(x)$ is continuous for all reals.
III. $f(x)$ is differentiable for all reals.

a) I only b) II only c) III only d) II and III only e) I, II, III

53) The domain of the function $f(x)$ graphed on the right is:

a) $[-2, 5]$ b) $[-3, 6)$ c) $[-2, \infty)$
d) $[-2, 5)$ e) $[-2, 6)$



Cal 2nd Exam

Definition of Continuity and Intermediate Value Theorem (*IVT*)

J Mumaugh

Ok.... this is old news... but still --- just to be sure we all are using the same definitions....

Def of Continuity: $\longrightarrow$ is a 3 step process! $\lim_{x\to a} f(x) = L \quad = \quad f(a)$ (2) (3) (1)

(1)
$f(a)$ exists

$f(a)$ must exist or the function is not continuous at $x = a$.

(2)
$\lim_{x\to a} f(x) = L$

Note: This implies that

$\lim_{x\to a^-} f(x) = \ell_1$

and

$\lim_{x\to a^+} f(x) = \ell_2$

and

$\ell_1 = \ell_2 = L$

(3)
$L = f(a)$

If (3) is true, then f is continuous at $x = a$.

Question: When do you *have* to approach from each side?
Answer: When the function changes in its nature or description at that point.

Example:

Given $f(x) = |x-2|$. Is $f(x)$ Continuous at $x = 5$? Sure it is. AND at x = 5 nothing about $f(x)$ changes so you would *not* need to show the approach from each side.

Example:

Given $f(x) = |x-2|$. Is $f(x)$ Continuous at $x = 2$? Prove it.

(1)
$f(2) = 0$

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} -(x-2) = 0 = \ell_1$

and

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} (x-2) = 0 = \ell_2$

and

$\ell_1 = \ell_2 = 0$

(2)
$\lim_{x\to a} f(2) = 0$

(3)
$0 = 0$

So $f(x)$ is continuous at $x = 2$.

Thm: If a function is differentiable, then the function is continuous.

Proof:

Assume: $\lim_{x\to a} f(x) = f(a)$ is true.

$$\lim_{x\to a} f(x) - f(a) = 0$$

$$\lim_{x\to a} f(x) - f(a)\cdot\frac{(x-a)}{(x-a)} = 0$$

$$\lim_{x\to a} \frac{f(x)-f(a)}{(x-a)}\cdot(x-a) = 0$$

$$\underbrace{\lim_{x\to a} \frac{f(x)-f(a)}{(x-a)}}\cdot(x-a) = 0$$

$$f'(a)\cdot 0 \quad = 0$$

$\uparrow$ given as existing

Thus, if $f'(a)$, ie. if f has a derivative at $x = a$, then f is continuous at $x = a$.

Is this an example of "assuming what we are trying to prove? Yes and it is allowable as long as we understand the significance of the possible results.

If we end with a "truth condition" then we have, in fact, proved the theorem.

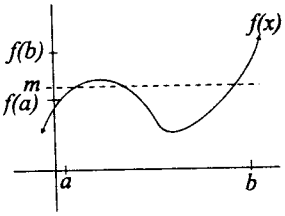
If we end with a "false condition" then we have proven nothing. That is we have neither proved the theorem to be true or false.

The Intermediate Value Theorem

Let f be continuous in [a, b]. Let m be a number such that $f(a) \le m \le f(b)$ (or vice-versa) then there exists at X such that $f(X) = m$.

The car speed is 25 mph and then later it is 50 mph. Did it ever read 40 mph? Of course. How do you know? We know because the speed changes are continuous so to get from 25 to 50, it had to read 30 and 35 as well as 40mph.

Graphically:



Note: m is a range item which is between $f(a)$ and $f(b)$. In this example, there are three values which could be X.

We used the IVT in Analysis. We found $f(2) = 5$ and $f(3) = -4$ so we knew there was a root between 2 and 3. ie. $m = 0$, $a = 2, b = 3$, and X might have been $\frac{11}{5}$.

It must be time to enjoy.

Degrees vs Radians!

[redacted]
From: Jim [teeitup98@telocity.com]
Sent: Friday, January 18, 2002 8:29 PM
To: Jim Mumaugh - Vaca High
Subject: FW: [ap-calculus] Re:sin x/ x

This is an excellent explanation concerning Sine (degrees) vs Sine (radians)

[redacted]
Reply-To: "AP-Calculus" <ap-calculus@list.collegeboard.org>
Date: Fri, 18 Jan 2002 15:01:05 -0700 (MST)
To: "AP-Calculus" <ap-calculus@list.collegeboard.org>
Cc: marain@rih.org
Subject: [ap-calculus] Re:sin x/ x

[redacted]

> Yes, we're flogging the horse, but, IMHO, if our goal to help our students develop PUFM (love those silly acronyms!) at the level of calculus and beyond, we need to clarify this whole issue.
> Again, as I stated on 14 Jan (pls refer to this), the DEFINITION (as it is given in most graduate texts of analysis) or the series expansion for sin x is
> x - x^3/3! + x^5/5! - ...
> This expression for sin x has nothing to do with how the input x is interpreted, since x could represent time in seconds! Students need only to accept here that x is a real number.
> However, if one wants to USE this representation of sin x to determine the sine of an angle, then the angle must be in radians to determine the correct answer.

This is one (correct) way to resolve the difficulties that folks have been having. I prefer a slightly different way of looking at things:

The sine function can be correctly defined in any of several ways:

1. sin x = x - x^3/3! + x^5/5! - ... .

2. The sine function is the inverse of the function

u |--> Integrate[1/Sqrt[1 - t^2], {t, 0, u}]

for -1 <= u <= 1 (the integral is improper, convergent in the cases of equality) with extension to the real line by appropriate trigonometric identities.

3. The sine function is given by the usual "wrapping" function—which, when properly done, involves lengths—but **not** angles.

4. The function x |--> sin x is the unique solution of the initial value problem y" + y = 0, y[0] = 0, y'[0] = 1.

The sine function, defined in any of these equivalent manners, satisfies the condition Limit[sin[x]/x, x -> 0] = 1, which is most easily seen by reference to definition 1 above. (Of course, definition 1 becomes a theorem if one chooses to use one of the other definitions.) This sine

function also turns out to be exactly we need for solving triangles when their angles are measured in radians.

So far, so good. The point that had been missed by most of those who've been puzzled about what happens when we measure angles in degrees is this: Although we write "sin A" for the sine of an angle A given in degrees, the symbol "sin" does not stand for the sine function given by any of the definitions given above. Instead, it stands for an altogether new function, DegreeSine, whose definition is

DegreeSine[A] = sin[A Pi/180], (*)

where the "sin" on the right side of (*) -is- the original sine function given by any of 1--4. When you change mode on your calculator, you are switching back and forth between the original sine function and the DegreeSine function—though nobody ever seems to acknowledge this. Consequently,

Limit[DegreeSine[A]/A , A ->0]

= Limit[sin[A Pi/180]/A, A -> 0]

= (Pi/180) Limit[sin[A Pi/180]/(A Pi/180), A -> 0]

= Pi/180.

--Lou Talman
Department of Mathematical and Computer Sciences
Metropolitan State College of Denver

http://clem.mscd.edu/~talmanl

So far, so good. The point that had been missed by most of those who've been puzzled about what happens when we measure angles in degrees is this: Although we write "sin A" for the sine of an angle A given in degrees, the symbol "sin" DOES NOT stand for the sine function givenby any of the definitions given above. Instead, it stands for an altogether new function, DegreeSine[A] = sin [A pi/180]. (*)

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leave-ap-calculus-5961519l@list.collegeboard.org

Jim Mumaugh - Vaca High

From: Jim [teeitup98@telocity.com]
Sent: Monday, January 21, 2002 7:47 PM
To: Jim Mumaugh - Vaca High
Subject: FW: [ap-calculus] Re:sin x/ x

From: LnMcmullin@aol.com
Reply-To: "AP-Calculus" <ap-calculus@list.collegeboard.org>
Date: Mon, 21 Jan 2002 08:12:28 EST
To: "AP-Calculus" <ap-calculus@list.collegeboard.org>
Subject: [ap-calculus] Re:sin x/ x

In a message dated 1/20/2002 6:41:34 PM Eastern Standard Time, huffhaus@pacbell.net writes:

> Lou, doesn't the 'wrapping' function imply wrapping around a Unit Circle, so that the lengths are equivalent to angles in radians?

Lengths (linear measure even if curved) cannot be equal to an angle measure. An (central) angle of one radian is the angle subtended by an arc whose length is equal to the radius of the circle. In general the arc length (s) , angle (theta) and radius (r) are related by the equation s = r* (theta) as I'm sure you know. Do the unit analysis: since s and r are length units, theta has no units - a scalar. It's called "radian" as a convenience when speaking.

Lin

Lin McMullin
Niantic, CT. ----------

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To unsubscribe send a blank email to leave-ap-calculus-

[redacted]
From: Jim [teeitup98@telocity.com]
Sent: Monday, January 21, 2002 7:49 PM
To: Jim Mumaugh - Vaca High
Subject: FW: [ap-calculus] Re:sin x/ x; find the error

[redacted]
Reply-To: "AP-Calculus" <ap-calculus@list.collegeboard.org>
Date: Sun, 20 Jan 2002 21:14:01 -0500
To: "AP-Calculus" <ap-calculus@list.collegeboard.org>
Cc: "AP-Calculus" <ap-calculus@list.collegeboard.org>, marain@rih.org
Subject: [ap-calculus] Re:sin x/ x; find the error

On 01/20/2002 09:53:54 AM Jerry Uhl wrote:

> To solve y'[x] = Sqrt[1 - y[x]^2] with y[0] = 0, calculus books do this:
>
> ** y'[x]/Sqrt[1 - y[x]^2] = 1
>
> ** ArcSin[y[x]] = x + C.
>
> ** C = 0 because y[0] = 0.
>
> ** y[x] = Sin[x]
>
> So far so good.
>
> But the trouble is that this y[x] cannot solve
>
> y'[x] = Sqrt[1 - y[x]^2].
>
> Reason: y'[x] = Sqrt[1 - y[x]^2]. forces y'[x] to be non-negative and this is not true for y[x] = Sin[x].
>
> -Jerry Uhl
> --
> ---
> Jerry Uhl

Jerry,

Your're starting to repeat yourself—you brought this very question up last year!

BTW, Lou Talman did not make the mistake that you are asking people to find above. (Nor am I saying that you claimed he did.) In his post he clearly says one should extend the function by using appropriate identities.

Doug

P.S. The error above is going from ArcSin[y[x]] = x to y[x] = Sin[x].

Jim Mumaugh - Vaca High
From: Jim [teeitup98@telocity.com]
Sent: Monday, January 21, 2002 7:49 PM
To: Jim Mumaugh - Vaca High
Subject: FW: [ap-calculus] Re:sin x/ x; find the error

From: Dave Marain <marain@nlh.org>
Reply-To: "AP-Calculus" <ap-calculus@list.collegeboard.org>
Date: 20 Jan 02 22:38:08 -0500 (EST)
To: "AP-Calculus" <ap-calculus@list.collegeboard.org>
Subject: [ap-calculus] Re:sin x/ x; find the error

Jerry,
This is a wonderful exercise for my students to review important domain/range issues for ArcSin, etc.
My thinking is that the original diff eq restricts y such that -1<=y<=1. Then, ArcSin[y(x)] = x, imposes the restriction -pi/2<=x<=pi/2, as it should.
Thus, the solution y = Sin(x) to the differential equation is only valid on this neighborhood of 0. Furthermore, y'(x) = cos(x) certainly is nonnegative on this restricted domain, so we haven't contradicted the sign of the principal square root Sqrt(1-y^2). Did I 'find the error'?!?
Dave Marain

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From: Jim [teeitup98@telocity.com]
Sent: Sunday, January 20, 2002 8:54 AM
To: Jim Mumaugh - Vaca High
Subject: FW: [ap-calculus] Re:sin x/ x; find the error

From: Jerry Uhl <juhl@cm.math.uiuc.edu>
Reply-To: "AP-Calculus" <ap-calculus@list.collegeboard.org>
Date: Sat, 19 Jan 2002 23:30:32 -0600
To: "AP-Calculus" <ap-calculus@list.collegeboard.org>
Cc: marain@rih.org
Subject: [ap-calculus] Re:sin x/ x; find the error

To solve y'[x] = Sqrt[1 - y[x]^2] with y[0] = 0, calculus books do this:

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** ArcSin[y[x]] = x + C.

** C = 0 because y[0] = 0.

** y[x] = Sin[x]

So far so good.

But the trouble is that this y[x] cannot solve

y'[x] = Sqrt[1 - y[x]^2].

Reason: y'[x] = Sqrt[1 - y[x]^2]. forces y'[x] to be non-negative and
this is not true for y[x] = Sin[x].

-Jerry Uhl
--

Jerry Uhl juhl@cm.math.uiuc.edu
Professor of Mathematics, Professor of Education
University of Illinois at Urbana-Champaign

Calculus&Mathematica, Vector Calculus&Mathematica, DiffEq&Mathematica,
Matrices,Geometry&Mathematica, ProbStat&Mathematica, NetMath

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"Science is built up of facts, as a house is built of stones; but an
accumulation of facts is no more a science than a heap of stones is a
house."
-----Henri Poincare

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$$\lim_{x \to \infty} \left(\sqrt{x^2 + x} - \sqrt{x^2 + 4} \right)$$

$$\lim_{x \to \infty} \left(\sqrt{x^2 + x} - \sqrt{x^2 + 4} \right)$$

$$\lim_{x \to \infty} \left(x\sqrt{1 + \frac{1}{x}} - x\sqrt{1 + \frac{4}{x^2}} \right)$$

$$\lim_{x \to \infty} x\left(\sqrt{1 + \frac{1}{x}} - \sqrt{1 + \frac{4}{x^2}} \right)$$

$$\lim_{x \to \infty} \left(\sqrt{x^2 + x} - \sqrt{x^2 + 4} \right) \frac{\sqrt{x^2 + x} + \sqrt{x^2 + 4}}{\sqrt{x^2 + x} + \sqrt{x^2 + 4}}$$

$$\lim_{x \to \infty} \left(\frac{x^2 + x - x^2 - 4}{\sqrt{x^2 + x} + \sqrt{x^2 + 4}} \right)$$

$$\lim_{x \to \infty} \left(\frac{x\left(1 - \frac{4}{x}\right)}{x\left(\sqrt{1 + \frac{1}{x}} + \sqrt{1 + \frac{4}{x^2}} \right)} \right)$$

$$\lim_{x \to \infty} \left(\frac{\left(1 - \frac{4}{x}\right)}{\left(\sqrt{1 + \frac{1}{x}} + \sqrt{1 + \frac{4}{x^2}} \right)} \right)$$

$$\infty \cdot \left(\sqrt{1} - \sqrt{1} \right) = \frac{1 - 0}{\sqrt{1} + \sqrt{1}}$$

$$\infty \cdot 0 = \frac{1}{2}$$

Obviously ...

Calculus 2nd Exam $\dfrac{\sin x}{x}$ J. Mumaugh

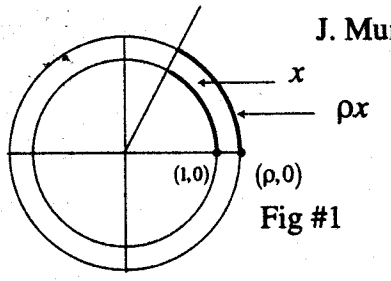

Fig #1

We know that the arc lengths in figure #1 are x and ρx.

We also know that the circumference of a circle is $2\pi r$ and the area is πr^2.

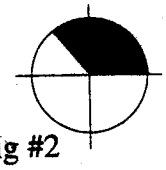
fig #2

ρ is a less common name for the radius, r.
Let A be the area of the shaded part of this circle.

This should be reasonable: $\dfrac{\text{Shaded Area}}{\text{Area of Circle}} = \dfrac{A}{\pi r^2} = \dfrac{\theta r}{2\pi r} = \dfrac{\text{Arc Length of Shaded Part}}{\text{Total Arc Length of Circle}}$

Let's assume that we are interested in the unit circle only.... then

$\dfrac{\text{Shaded Area}}{\text{Area of Circle}} = \dfrac{A}{\pi} = \dfrac{\theta}{2\pi} = \dfrac{\text{Arc Length of Shaded Part}}{\text{Total Arc Length of Circle}}$

$\dfrac{A}{\pi} = \dfrac{\theta}{2\pi}$

$A = \dfrac{\theta}{2}$... thus the area of the sector of a circle of a unit circle is $\dfrac{\theta}{2}$.

Consider.. only the 1st quadrant of a circle... arc length is $\dfrac{\pi}{2}$... divided by 2 gives $\dfrac{\pi}{4}$ which is the area of one quarter of a unit circle. The area trapped by arc length $\dfrac{\pi}{3}$ (60 degrees) is $\dfrac{\pi}{6}$.

The general formula for the area of a sector of a circle is $\dfrac{r\theta}{2}$ where r is the radius of the circle and θ is the arc length in radians.

One final note before we begin the lesson for today... the text uses θ as a *radian* measure rather than as a degree measure. I generally prefer to use x for a radian measure. I use the θ here only to be consistent with the text but remember that we are talking about a radian measurement.

WTP $\lim\limits_{\theta \to 0} \dfrac{\sin\theta}{\theta} = 1$ $\qquad 0 \le \theta < \dfrac{\pi}{2}$

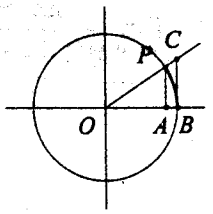

fig #3
$P\ (\cos\theta, \sin\theta)$

$$\text{Area of } \triangle OAP \le \text{Area of Sector } \widehat{OBP} \le \text{Area of } \triangle OBC$$

$$\frac{1}{2}\sin\theta\cos\theta \le \frac{\theta}{2} \le \frac{1}{2} \quad \overline{OB} \quad \overline{BC}$$

$$\frac{1}{2}\sin\theta\cos\theta \le \frac{\theta}{2} \le \frac{1}{2} \quad (1) \quad \frac{\sin\theta}{\cos\theta}$$

Show the similar triangles which lead to this substitution.

$$\sin\theta\cos\theta \le \theta \le \frac{\sin\theta}{\cos\theta}$$

$$\cos\theta \le \frac{\theta}{\sin\theta} \le \frac{1}{\cos\theta}$$

Now take the limit of the whole thing as θ goes to zero.....

$$\lim_{\theta\to 0}\left(\cos\theta \le \frac{\theta}{\sin\theta} \le \frac{1}{\cos\theta}\right)$$

$$\left(1 \le \lim_{\theta\to 0}\frac{\theta}{\sin\theta} \le \frac{1}{1}\right)$$

A reasonable person would notice that whatever $\lim_{\theta\to 0}\frac{\theta}{\sin\theta}$ is, it is between 1 and 1 so.... that reasonable person would conclude that $\lim_{\theta\to 0}\frac{\theta}{\sin\theta}$ is in fact 1. What does that mean?

For a fraction to be equal to 1 means that the ratio of the top to the bottom is 1 -- that they are actually equal. If $\lim_{\theta\to 0}\frac{\theta}{\sin\theta}$ is one then what would $\lim_{\theta\to 0}\frac{\sin\theta}{\theta}$ be? One, of course.
Thus the curved length θ is almost equal to the vertical length $\sin\theta$ when θ is small.

Consider... $\lim_{\theta\to 0}\frac{1-\cos\theta}{\theta}$

$$\lim_{\theta\to 0}\frac{1-\cos\theta}{\theta}\cdot\frac{1+\cos\theta}{1+\cos\theta}$$

$$\lim_{\theta\to 0}\frac{1-\cos^2\theta}{\theta(1+\cos\theta)}$$

$$\lim_{\theta\to 0}\frac{\sin^2\theta}{\theta(1+\cos\theta)} = \lim_{\theta\to 0}\frac{\sin\theta}{\theta}\cdot\frac{\sin\theta}{1}\cdot\frac{1}{(1+\cos\theta)}$$

$$= 1\cdot\frac{0}{1}\cdot\frac{1}{(1+1)} = 0$$

In this situation, the horizontal measure $1-\cos\theta$ goes to zero faster than the arc length θ.

Use the two identities $\lim_{\theta\to 0}\frac{1-\cos\theta}{\theta} = 0$ and $\lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1$ to simplify and compute the value of the limit problems in the homework.